

Section 3.2

3.) $f(x) = x^3 - 2x + 1$ at $x = 2$:

i.) $f(2) = 2^3 - 2(2) + 1 = 5$

ii.) $\lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} (x^3 - 2x + 1) = 8 - 4 + 1 = 5$

iii.) $\lim_{x \rightarrow 2} f(x) = 5 = f(2)$,

so f is continuous at $x = 2$

5.) $f(x) = \begin{cases} \frac{x^2 - x - 2}{x - 2} & \text{if } x \neq 2 \\ 3 & \text{if } x = 2 \end{cases}$ at $x = 2$:

i.) $f(2) = 3$

ii.) $\lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x - 2}$

$= \lim_{x \rightarrow 2} \frac{(x-2)(x+1)}{x-2} = 3$

iii.) $\lim_{x \rightarrow 2} f(x) = 3 = f(2)$,

so f is continuous at $x = 2$.

7.) $f(x) = \begin{cases} \frac{x^2 - 9}{x - 3} & \text{if } x \neq 3 \\ a & \text{if } x = 3 \end{cases}$;

$\lim_{x \rightarrow 3} f(x) = \lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = \lim_{x \rightarrow 3} \frac{(x-3)(x+3)}{x-3} = 6$

so choose $f(3) = a = 6$.

$$8.) f(x) = \begin{cases} \frac{x^2+x-2}{x-1} & \text{if } x \neq 1 \\ a & \text{if } x = 1 \end{cases}$$

i.) $f(1) = a$

ii.) $\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{x^2+x-2}{x-1}$

" $\frac{0}{0}$ " $\lim_{x \rightarrow 1} \frac{(x-1)(x+2)}{x-1} = 1+2 = 3$

iii.) Let $\boxed{a=3}$, then

$\lim_{x \rightarrow 1} f(x) = 3 = f(1)$ and f is continuous at $x=1$.

$$9.) f(x) = \begin{cases} \frac{1}{x-3} & \text{if } x \neq 3 \\ 0 & \text{if } x = 3 \end{cases} \quad \text{at } x=3:$$

i.) $f(3) = 0$

ii.) $\lim_{x \rightarrow 3} f(x) = \lim_{x \rightarrow 3} \frac{1}{x-3} = \frac{1}{0} = \pm \infty$ (not finite)

so f is not continuous at $x=3$.

10.) $f(x) = \frac{1}{x^2-1}$; $y=1$ is cont. for all

x -values (line) and $y=x^2-1$ is cont.

for all x -values (parabola). Thus, $f(x) = \frac{1}{x^2-1}$ is continuous (quotient)

for all x -values, except where

$x^2 - 1 = (x-1)(x+1) = 0$, i.e., except at $x=1$ and $x=-1$.

$$11.) f(x) = \begin{cases} \frac{x^2 - 3x + 2}{x - 2} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$$

Check $x=1$:

i.) $f(1) = 1$

ii.) $\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x - 2} = \frac{1 - 3 + 2}{1 - 2} = 0$

so f is NOT continuous at $x=1$;
in addition, $y = x^2 - 3x + 2$ and $y = x - 2$
are both continuous for all values
of x (both are polynomials), so

$y = \frac{x^2 - 3x + 2}{x - 2}$ is continuous for
all values of x (quotient), except
where $x - 2 = 0$, i.e., except at $x=2$.

$$12.) f(x) = \begin{cases} x^2 - 1 & \text{if } x \leq 0 \\ x & \text{if } x > 0 \end{cases} \quad \text{at } x=0:$$

i.) $f(0) = 0^2 - 1 = 0 - 1 = -1$

ii.) $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} x = 0$ and

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (x^2 - 1) = -1, \text{ so}$$

$\lim_{x \rightarrow 0} f(x)$ DNE; thus, f is
not continuous at $x=0$;

note that $y = x^2 - 1$ (parabola)
is continuous for all $x < 0$ and
 $y = x$ (line) is continuous for
all $x > 0$.

worksheet 1

1.) a.) $f(x) = \begin{cases} x^2 + 3 & \text{if } x \neq -1 \\ 2 & \text{if } x = -1 \end{cases} ; \text{ at } x = -1 :$

i.) $f(-1) = 2$

ii.) $\lim_{x \rightarrow -1} f(x) = \lim_{x \rightarrow -1} (x^2 + 3) = 1 + 3 = 4$

iii.) $\lim_{x \rightarrow -1} f(x) = 4 \neq 2 = f(-1)$ so

f is NOT continuous at $x = -1$

b.) $g(x) = \begin{cases} x+1 & \text{if } x \geq 0 \\ 2-x^2 & \text{if } x < 0 \end{cases} ; \text{ at } x = 0 :$

i.) $g(0) = 0 + 1 = 1$

ii.) $\lim_{x \rightarrow 0^+} g(x) = \lim_{x \rightarrow 0^+} (x+1) = 1 ;$

$\lim_{x \rightarrow 0^-} g(x) = \lim_{x \rightarrow 0^-} (2-x^2) = 2 ;$ so

$\lim_{x \rightarrow 0} g(x)$ DNE ; thus

g is NOT continuous at $x = 0$

c.) $f(x) = \begin{cases} x-2 & \text{if } x > 1 \\ 0 & \text{if } x = 1 \\ -x & \text{if } x < 1 \end{cases} ; \text{ at } x = 1 :$

i.) $f(1) = 0$

ii.) $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (x-2) = -1 ;$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (-x) = -1 ; \text{ so}$$

$$\lim_{x \rightarrow 1} f(x) = -1$$

$$\text{iii.) } \lim_{x \rightarrow 1} f(x) = -1 \neq 0 = f(1), \text{ so}$$

f is NOT continuous at $x=1$.

2.) a.) $f(x) = x^5 + x^4 + x^3 + x^2 + x + 1$ is continuous for all values of x since it is a polynomial.

b.) $g(x) = \frac{\sin x}{x^2 + 4}$; $y = \sin x$ (well known) and $y = x^2 + 4$ (polynomial) are continuous for all values of x ; so the QUOTIENT

$g(x) = \frac{\sin x}{x^2 + 4}$ is continuous for all values of x since $x^2 + 4 \neq 0$.

c.) $f(x) = \frac{x+3}{x^2-4}$; $y = x+3$ and $y = x^2-4$ (polynomials) are continuous for all values of x ; the QUOTIENT

$f(x) = \frac{x+3}{x^2-4}$ is continuous for

all values of x except where $x^2 - 4 = 0$, i.e., except when $x = \pm 2$.

d.) $g(x) = \cos(x^3 - x)$; $f(x) = \cos x$ (well known) and $k(x) = x^3 - x$ (polynomial) are continuous for all values of x ; so the COMPOSITION $f(k(x)) = f(x^3 - x) = \cos(x^3 - x)$ is continuous for all values of x .

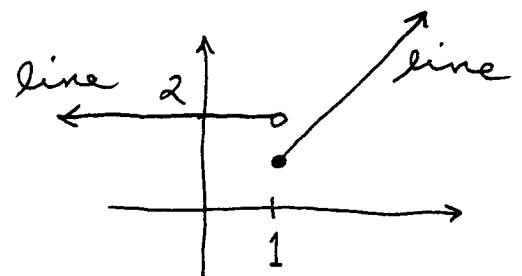
3.) a.) $f(x) = \begin{cases} \frac{x^2 - 7x + 6}{x - 6} & \text{if } x \neq 6 \\ A & \text{if } x = 6 \end{cases};$
 $\lim_{x \rightarrow 6} f(x) = \lim_{x \rightarrow 6} \frac{x^2 - 7x + 6}{x - 6} = \lim_{x \rightarrow 6} \frac{(x-6)(x-1)}{x-6}$
 $= 6 - 1 = 5$ and $f(6) = A$ so
 choose $A = 5$

b.) $f(x) = \begin{cases} A^2 x - A & \text{if } x \geq 1 \\ 2 & \text{if } x < 1 \end{cases};$

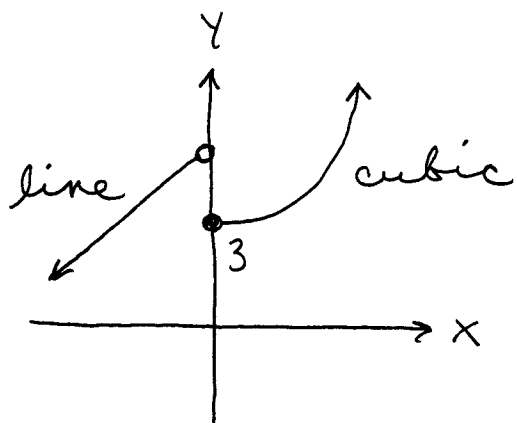
Require that
 $\lim_{x \rightarrow 1^+} f(x) = 2 \rightarrow$

$\lim_{x \rightarrow 1^+} (A^2 x - A) = 2 \rightarrow$

$A^2 - A = 2 \rightarrow A^2 - A - 2 = 0 \rightarrow$
 $(A-2)(A+1) = 0 \rightarrow A = 2 \text{ or } A = -1$



c.) $f(x) = \begin{cases} \frac{A+x}{A+1} & \text{if } x < 0 \\ Ax^3 + 3 & \text{if } x \geq 0 \end{cases};$



Require that
 $\lim_{x \rightarrow 0^-} f(x) = 3 \rightarrow$

$$\lim_{x \rightarrow 0^-} \frac{A+x}{A+1} = 3 \rightarrow$$

$$\frac{A+0}{A+1} = 3 \rightarrow A = 3A + 3 \rightarrow -3 = 2A \rightarrow$$

$$A = -3/2$$

$$d.) f(x) = \begin{cases} 3 & \text{if } x \leq 1 \\ Ax^2 + B & \text{if } 1 < x \leq 2 \\ 5 & \text{if } x > 2 \end{cases}$$

Require Two
 conditions:

$$i.) \lim_{x \rightarrow 1^+} f(x) = 3 \rightarrow$$

$$\lim_{x \rightarrow 1^+} (Ax^2 + B) = 3 \rightarrow$$

$$A + B = 3$$

AND

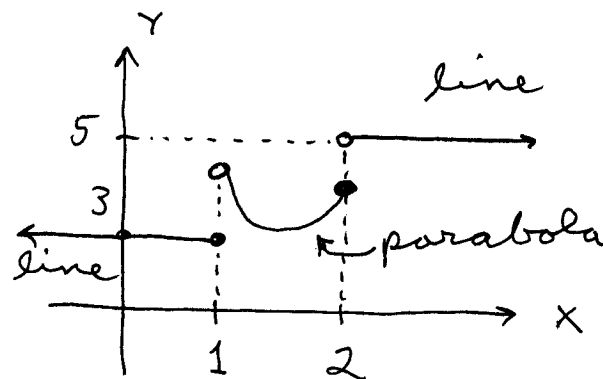
$$ii.) \lim_{x \rightarrow 2^-} f(x) = 5 \rightarrow \lim_{x \rightarrow 2^-} (Ax^2 + B) = 5 \rightarrow$$

$$4A + B = 5$$

; solve system:

$$B = 3 - A \rightarrow (\text{sub } B) \rightarrow 4A + (3 - A) = 5 \rightarrow$$

$$3A = 2 \rightarrow A = 2/3, B = 7/3$$



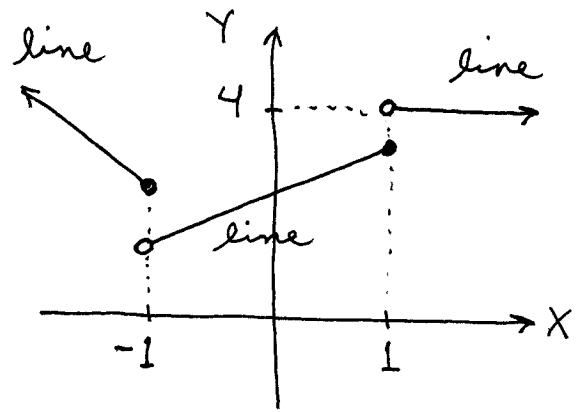
$$e.) f(x) = \begin{cases} Ax - B & \text{if } x \leq -1 \\ 2x + 3A + B & \text{if } -1 < x \leq 1 \\ 4 & \text{if } x > 1 \end{cases}$$

Require Two
conditions :

$$i.) \lim_{x \rightarrow 1^-} f(x) = 4 \rightarrow$$

$$\lim_{x \rightarrow 1^-} (2x + 3A + B) = 4 \rightarrow$$

$$2 + 3A + B = 4 \rightarrow \boxed{3A + B = 2} \text{ AND}$$



$$ii.) \lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^+} f(x) \rightarrow$$

$$\lim_{x \rightarrow -1^-} (Ax - B) = \lim_{x \rightarrow -1^+} (2x + 3A + B) \rightarrow$$

$$-A - B = -2 + 3A + B \rightarrow 2 = 4A + 2B \rightarrow$$

$$\boxed{2A + B = 1} ; \text{ solve system :}$$

$$3A + B = 2 \rightarrow B = 2 - 3A \rightarrow (\text{sub}) \rightarrow$$

$$2A + (2 - 3A) = 1 \rightarrow -A = -1 \rightarrow$$

$$\boxed{A = 1, B = -1}$$