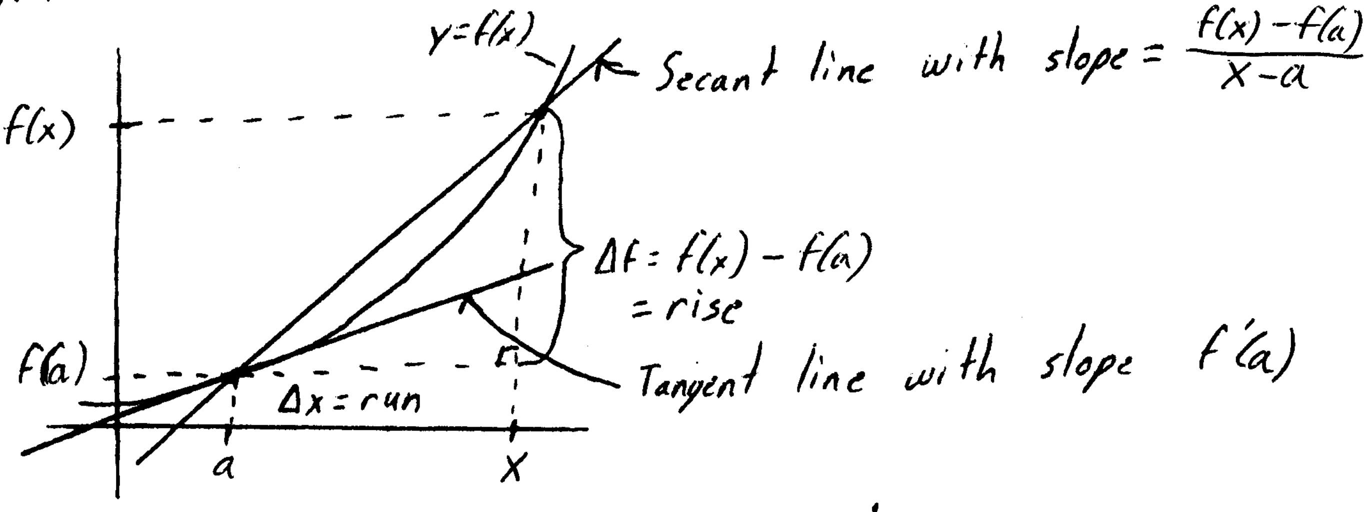
Math 125A

Vogler

The Derivative



Note that the slope of the secant line is

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{f(x)-f(a)}{x-a}$$

We can obtain slope of tangent line by letting Δx get 'small' (i.e. $x \to a$) which leads to the following:

<u>Defn</u> Let f be a function on (b,c). The <u>derivative</u> of f at the point $a \in (b,c)$ is

$$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

provided this limit exists and is finite.

- Notes: 1) If $f'(a)$ exists and is finite, we say f is <u>differentiable</u> at a. Otherwise, f is not differentiable at a.
  2) $f'(a)$ is the slope of the tangent line through point $(a, f(a))$.
  3) $\displaystyle\lim_{x\to a} \frac{f(x)-f(a)}{x-a} \approx \frac{\Delta f}{\Delta x}$ can be thought of as 'infintesmal division' (i.e. division of two 'small' numbers)
  4) Other 'disguises' include: $f'(a) = \displaystyle\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{f(a+h)-f(a-h)}{h}$.

<u>Thm</u> 28.2

If f is differentiable at a, then f is continuous at a.

- Moral: The set of differentiable functions is a subset of continuous functions.

Thm (28.3)

Let f & g be differentiable functions at $x=a$. Then the functions cf (with c a constant), $f+g$, fg, and $\frac{f}{g}$ are differentiable at $x=a$ when defined. The derivaties are:

1) $(cf)'(a) = c \cdot f'(a)$ (Constant Multiple Rule)
2) $(f+g)'(a) = f'(a) + g'(a)$ (Sum Rule)
3) $(fg)'(a) = f'(a)\,g(a) + f(a)\,g'(a)$ (Product Rule)
4) $\left(\frac{f}{g}\right)'(a) = \frac{[f'(a)g(a) - f(a)g'(a)]}{[g(a)]^2}$ (Quotient Rule)

Note: Clearly the quotient rule only makes sence if $g(a) \neq 0$.

Chain Rule (28.4)

If f is differentiable at a and g is differentiable at $f(a)$, then $(g \circ f)$ is differentiable at a. Moreover, $(g \circ f)'(a) = g'(f(a)) \cdot f'(a)$.

Derivative Rules for Well-Known Functions

1) $(x^n)' = nx^{n-1}$ for $n \in \mathbb{R}$ (Power Rule)
2) $(c)' = 0$ for $c \in \mathbb{R}$
3) $(\sin x)' = \cos x$
4) $(\cos x)' = -\sin x$
5) $(b^x)' = b^x \ln b$ for $b \in \mathbb{R}$
6) $(\log_b x)' = \frac{1}{x \ln b}$ for $b \in \mathbb{R}$
7) $(\tan x)' = \sec^2 x$
8) $(\cot x)' = -\csc^2 x$
9) $(\sec x)' = \sec x \tan x$
10) $(\csc x)' = -\csc x \cot x$

Note: You may use these facts on a test unless specifically asked to prove it using the definition of the derivative.