Math 125A
Vogler

The Mean Value Theorem

Interior Extrema Thm (29.1)

If f is differentiable at x_0 and obtains it's maximum or minimum value at x_0, then $f'(x_0)=0$.

-Note: This fact gives us a systematic way to find a max or min value by searching for x_0's where $f'(x_0)=0$.

Rolle's Thm (29.2)

Let f be continuous on $[a,b]$, differentiable on (a,b), and $f(a)=f(b)$. Then $\exists c \in (a,b)$ such that $f'(c)=0$.

Mean Value Thm (29.3)

Let f be continuous on $[a,b]$ and differentiable on (a,b). Then $\exists\, c \in (a,b)$ such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}.$$

-Notes: 1) Both theorems guarantee at least one c. There may be many or an infinite number of c's.

2) Rolle's Thm is a special case of the Mean Value Thm.

-Moral: For any differentiable function on a given interval, there exists a tangent line within the interval that has the same slope as the secant line across the interval.

Cor (29.4)

Let f be differentiable on (a,b) where $f'(x)=0 \ \forall x \in (a,b)$. Then f is a constant function on (a,b).

Cor (29.5)
Let f and g be differentiable functions with $f' = g'$ on (a,b). Then $\exists C \in \mathbb{R}$ such that $f(x) = g(x) + C$ $\forall x \in (a,b)$.

-Note: This fact guarantees that antiderivatives (a.k.a. integrals) are unique up to a constant.

Defn Let f be defined on interval I.

1) f is increasing on I if $\forall x_1, x_2 \in I$ w/ $x_1 < x_2 \Rightarrow f(x_1) \le f(x_2)$.
2) f is decreasing on I if $\forall x_1, x_2 \in I$ w/ $x_1 < x_2 \Rightarrow f(x_1) \ge f(x_2)$.

-Notes: 1) If f is only increasing or decreasing, we say f is monotone.
2) If there's a strict inequality ($<$ or $>$), we say f is strictly increasing of strictly decreasing, respectively.

Cor (29.7)
Let f be differentiable function on (a,b). Then

1) if $f'(x) > 0$ $\forall x \in (a,b) \Rightarrow f$ is strictly increasing.
2) if $f'(x) < 0$ $\forall x \in (a,b) \Rightarrow f$ is strictly decreasing.
3) if $f'(x) \ge 0$ $\forall x \in (a,b) \Rightarrow f$ is increasing.
4) if $f'(x) \le 0$ $\forall x \in (a,b) \Rightarrow f$ is decreasing.

-Note: This fact enables us to graph functions without a computor.

Intermediate Value Thm for Derivatives (29.8)
Let f be differentiable on (a,b). When $a < x_1 < x_2 < b$ and m lies between $f'(x_1)$ and $f'(x_2)$, $\exists c \in (x_1, x_2)$ such that $f'(c) = m$.

Thm (29.9)
Let f be 1-1 continuous function on open interval I and $J = f(I)$. If f is differentiable at $x_0 \in I$ and $f'(x_0) \neq 0$, then f^{-1} is differentiable at $y_0 = f(x_0) \in J$ and

$$(f^{-1})'(y_0) = \frac{1}{f'(x_0)}.$$