Math 125A
Voyler

Power Series

Defn Let $\{a_n\}$ be a sequence and x a variable. Then $\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$

is called a power series.

-Note: One can replace x with $(x-a)$ to obtain the power series centered at $x=a$.

Thm (23.1)
For the power series $\sum_{n=0}^{\infty} a_n x^n$, let

$$\beta = \limsup_{n\to\infty} |a_n|^{1/n} \quad \text{and} \quad R = \frac{1}{\beta}$$

Then a) the power series converges for $|x|<R$.
b) the power series diverges for $|x|>R$.

-Notes: 1) If $\beta=0$, we set $R=\infty$. If $\beta=\infty$, we set $R=0$.
2) R is called the radius of convergence.
3) Most times it's easier to compute $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$ (i.e ratio test). Corollary 12.3 tells us $\beta = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$.
4) There are exactly 3 scenerios for a power series:
a) it converges $\forall x \in \mathbb{R}$ (i.e. $R=\infty$)
b) it converges for only $x=0$ (i.e $R=0$)
c) it converges for x in a bounded interval centered at 0 (i.e. R is finite). This interval may be open, half-open, or closed, and it is called the interval of convergence.