18.2) It would breakdown because the limit of subsequences x_0 and y_0 does not have to be in the interval (a, b). Since we would only assume continuity of f on (a, b), we would lose it at the endpoints a and b. This could cause the function to be unbounded and not contain it's minimum and/or maximum. For example, take $f(x) = \dfrac{1}{x}$ on $(0, 1)$.

18.4) Suppose $S \subseteq \mathbb{R}$ and there exists a sequence $\{x_n\}$ in S that converges to $x_0 \notin S$. Consider the function $f(x) = \dfrac{1}{x - x_0}$ on S. f is continuous on S since $x_0 \notin S$ by the fact $x - x_0$ is a polynomial and the division law of continuity (Theorem 17.4). f is unbounded because of the division by zero that occurs within the sequence $f(x_n)$ (Theorem 9.10 and Exercise 9.10b in the case the sequence is negative).

18.6) We can rewrite the equation $x = \cos x$ as $x - \cos x = 0$. Let $f(x) = x - \cos x$ and $m = 0$. Notice that $f(0) = -1 < 0$ and $f(\frac{\pi}{2}) = \frac{\pi}{2} > 0$. Choose interval $(0, \frac{\pi}{2})$, so that $f(0) < m = 0 < f(\frac{\pi}{2})$. By the Intermediate Value Theorem, there exists at least one $x \in (0, \frac{\pi}{2})$ with $f(x) = m = 0$. With this x, we have

$$f(x) = 0 \Leftrightarrow x - \cos x = 0 \Leftrightarrow x = \cos x.$$

Hence, $x = \cos x$ for $x \in (0, \frac{\pi}{2})$.

18.8) Suppose f is a continuous function on $\mathbb{R}$ and $f(a)f(b) < 0$ for some $a, b \in \mathbb{R}$. Since $f(a)f(b) < 0$, one of the two values $f(a)$ or $f(b)$ must be positive while the other is negative. Without loss of generality, assume $f(a) < 0$ and $f(b) > 0$. Let $m = 0$ and choose interval (a, b) since $f(a) < m = 0 < f(b)$. By the Intermediate Value Theorem, there exists at least one $x \in (a, b)$ with $f(x) = m = 0$, proving the claim.

18.10) Suppose f is continuous on $[0, 2]$ and $f(0) = f(2)$. Define $g(x) = f(x+1) - f(x)$ on $[0, 1]$. Notice $g(0) = f(2) - f(1)$ and $g(1) = f(1) - f(0) = f(1) - f(2)$ by assumption. So we have $g(0) = -g(1)$. We consider two cases: a) $g(0) = 0$ or b) $g(0) \neq 0$.

Case a) If $g(0) = 0$, this implies $f(2) - f(1) = 0$ and consequently $f(2) = f(1)$. So for $x = 2$ and $y = 1$, we have $|x - y| = 1$ and $f(x) = f(y)$, proving the claim.

Case b) If $g(0) \neq 0$, without loss of generality we can assume $g(0) < 0$, which implies $g(1) > 0$. g is continuous since f is continuous and composition and differences preserve continuity. Let $m = 0$, and choose interval $(0, 1)$ since $g(0) < m = 0 < g(1)$. By the Intermediate Value Theorem, there exists at least one $c \in (0, 1)$ with $g(c) = m = 0$. With this c, we have

$$g(c) = 0 \Leftrightarrow f(c+1) - f(c) \Leftrightarrow f(c+1) = f(c).$$

So for $x = c + 1$ and $y = c$, we have $|x - y| = 1$ and $f(x) = f(y)$, proving the claim.