Math 21D
Vogler
Discussion Sheet 4

1.) Let R be the solid region bounded by the surfaces $z = \sqrt{4 - x^2 - y^2}$ and $z = 0$. SET UP BUT DO NOT EVALUATE triple integrals which represent the volume of the solid using spherical coordinates.

2.) Let R be the solid region bounded by the surfaces $z = \sqrt{x^2 + y^2}$ and $z = \sqrt{18 - x^2 - y^2}$. SET UP BUT DO NOT EVALUATE triple integrals which represent the volume of the solid using spherical coordinates.

3.) Let R be the solid region inside the surface $x^2 + y^2 = 4$ and bounded by the surfaces $z = 0$ and $z = \sqrt{9 - x^2 - y^2}$. SET UP BUT DO NOT EVALUATE triple integrals which represent the volume of the solid using spherical coordinates.

4.) Consider the chocolate chip cookie bounded by the surfaces $z = 9 - x^2 - y^2$ and $z = 9 - 3y$. The density of the cookie at point $P = (x, y, z)$ is given by one plus the distance from P to the point $(0, 0, 9)$. SET UP BUT DO NOT EVALUATE triple integrals which represent the cookie's total mass (yummy) using

a.) rectangular coordinates.
b.) cylindrical coordinates.
c.) spherical coordinates.

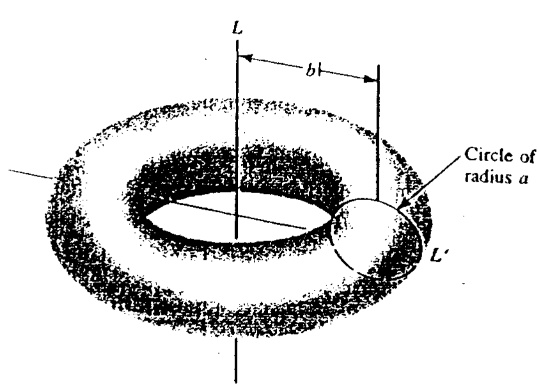
5.) Convert the following cylindrical integral to spherical coordinates. DO NOT EVALUATE THE INTEGRAL.

$$\int_0^{2\pi} \int_2^{\sqrt{5}} \int_0^{\sqrt{5-r^2}} r^2 z \cos\theta \, dz \, dr \, d\theta$$

6.) Sketch the solid whose volume is given by the following spherical integral.

$$\int_0^{\pi} \int_0^{\pi/2} \int_2^{3} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

7.) SET UP BUT DO NOT EVALUATE a triple integral which represents the volume of the given doughnut (torus).



4.) Plot the curve C determined by each vector function.

a.) $\vec{r}(t) = e^t\ \vec{i} + e^{3t}\ \vec{j}$ for $-1 \le t \le 1$

b.) $\vec{r}(t) = \cos t\ \vec{i} + \sin t\ \vec{j}$ for $0 \le t \le 2\pi$

c.) $\vec{r}(t) = \sqrt{t}\cos t\ \vec{i} + \sqrt{t}\sin t\ \vec{j}$ for $0 \le t \le 4\pi$

d.) $\vec{r}(t) = 2t\ \vec{i} + 3t\ \vec{j} + 4t\ \vec{k}$ for $0 \le t \le 2$

e.) $\vec{r}(t) = \sin t\ \vec{i} + \cos t\ \vec{j} + t\ \vec{k}$ for $0 \le t \le 4\pi$

5.) Assume that the motion of a particle along path C is determined by the position function $\vec{r}(t) = f(t)\ \vec{i} + g(t)\ \vec{j} + h(t)\ \vec{k}$. We know that the speed of motion at time t is $\left|\vec{v}(t)\right| = \dfrac{ds}{dt} = \sqrt{(f'(t))^2 + (g'(t))^2 + (h'(t))^2}$. Show that the acceleration of motion at time t is given by $a(t) = \dfrac{\vec{v}(t) \cdot \vec{a}(t)}{\left|\vec{v}(t)\right|}$.

6.) Assume that the path C of a bird in flight is determined by the vector function $\vec{r}(t) = t\ \vec{i} + t^2\ \vec{j} + 2t\ \vec{k}$ for $t \ge 0$. Find the bird's position vector, velocity vector, speed, acceleration vector, and acceleration at time

a.) $t = 0$.

a.) $t = 1$.

a.) $t = 2$.

7.) The position of a bicyclist is determined by the vector function $\vec{r}(t) = (3t)\ \vec{i} + (3\sin t)\ \vec{j}$ for $0 \le t \le 2\pi$. Determine the bicyclist's maximum speed.

8.) Find vector function $\vec{r}(t)$ if $\vec{r}''(t) = \vec{i} + t\ \vec{j} + \cos 2t\ \vec{k}$, $\vec{r}'(0) = \vec{i} + \vec{j} + \vec{k}$, and $\vec{r}(0) = 2\vec{i} - \vec{j} - \vec{k}$.

9.) A super ball is projected at an angle of 75^o with initial speed 100 $m./sec.$

a.) How high does the ball go ?

b.) How long is the ball in the air ?

c.) How far downrange does the ball travel ?

10.) A ball bearing is projected at an angle of 60^o and lands 500 feet downrange. What was the ball bearing's initial speed ?

11.) A kiwi is projected at an angle of α degrees with an initial speed of 100 $m./sec.$ If it lands 200 meters downrange, what is α ?

12.) Assume that $\vec{u}(t) = a(t)\ \vec{i} + b(t)\ \vec{j} + c(t)\ \vec{k}$, $\vec{v}(t) = f(t)\ \vec{i} + g(t)\ \vec{j} + h(t)\ \vec{k}$, and $y = k(t)$.

a.) (Dot Product Rule) Prove that $D\{\vec{u}(t) \cdot \vec{v}(t)\} = \vec{u}(t) \cdot \vec{v}'(t) + \vec{u}'(t) \cdot \vec{v}(t)$.

b.) (Chain Rule) Prove that $D\{\vec{u}(k(t))\} = \vec{u}'(g(t))k'(t)$

Thanks to Dr. Kouba