

Math 21D
Vogler
Discussion Sheet 6

1.) Find the work done by each force field $\vec{F}$ along the given path C .

a.) $\vec{F}(x, y) = (x^2y) \vec{i} + (y) \vec{j}$

i.) C : graph of $y = x^2$ for $0 \leq x \leq 2$

ii.) C : graph of $y = 2x$ for $0 \leq x \leq 2$

b.) $\vec{F}(x, y) = (x) \vec{i} + (y) \vec{j}$ and $C : \vec{r}(t) = (3 \cos t) \vec{i} + (4 \sin t) \vec{j}$ for $0 \leq t \leq 2\pi$

c.) $\vec{F}(x, y) = (y) \vec{i} + (yz) \vec{k}$

i.) $C : \vec{r}(t) = (\cos t) \vec{i} + (\sin t) \vec{j} + (3t) \vec{k}$ for $0 \leq t \leq 2\pi$

ii.) C : line segment from point $(0, 0, 0)$ to $(3, -1, 2)$

2.) Find the flux of each vector field $\vec{F}$ across the given closed path C .

a.) $\vec{F}(x, y) = (x^2) \vec{i} + (y^2) \vec{j}$ and C : circle $x^2 + y^2 = 1$

b.) $\vec{F}(x, y) = (xy) \vec{i} + (x^2) \vec{j}$ and C : ellipse $\left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$

c.) $\vec{F}(x, y) = (x) \vec{i} + (y^2) \vec{j}$ and C : the parabola $y = x^2$ from point $(0, 0)$ to $(1, 1)$, the line $y = 1$ from point $(1, 1)$ to $(0, 1)$, and the line $x = 0$ from point $(0, 1)$ to $(0, 0)$

3.) Find a scalar function f whose gradient field $\vec{\nabla} f$ is given.

a.) $\vec{\nabla} f(x, y) = 3 \vec{i} - 4 \vec{j}$

b.) $\vec{\nabla} f(x, y) = (y^2) \vec{i} + (2xy) \vec{j}$

c.) $\vec{\nabla} f(x, y) = (x^2ye^{xy} + 2xe^{xy}) \vec{i} + (x^3e^{xy}) \vec{j}$

d.) $\vec{\nabla} f(x, y, z) = (3x) \vec{i} + (4y) \vec{j} + (5z) \vec{k}$

e.) $\vec{\nabla} f(x, y, z) = (yz) \vec{i} + (xz) \vec{j} + (xy) \vec{k}$

f.) $\vec{\nabla} f(x, y, z) = \frac{2x}{x^2 + y^2 + z^2} \vec{i} + \frac{2y}{x^2 + y^2 + z^2} \vec{j} + \frac{2z}{x^2 + y^2 + z^2} \vec{k}$

g.) $\vec{\nabla} f(x, y) = (xy^2ze^{xz} + y^2e^{xz}) \vec{i} + (2xye^{xz}) \vec{j} + (x^2y^2e^{xz}) \vec{k}$

"In mathematics, you don't understand things. You just get used to them." – Johann von Neumann, a famous Hungarian mathematician