1.) Consider the flat region R lying inside the circle $x^2 + (y-2)^2 = 4$ and above the line $y = 2$. Sketch the region and describe R using

a.) vertical cross sections.

$$\begin{cases} -2 \le x \le 2, \\ 2 \le y \le 2 + \sqrt{4-x^2} \end{cases}$$

b.) horizontal cross sections.

$$\begin{cases} 2 \le y \le 4, \\ -\sqrt{4y-y^2} \le x \le +\sqrt{4y-y^2} \end{cases}$$

$x^2 + (y-2)^2 = 4 \rightarrow$

$y = 2 \pm \sqrt{4-x^2}$

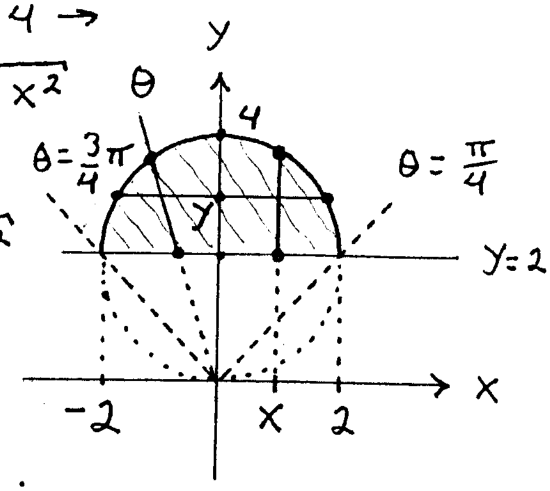
or

$x = \pm\sqrt{4-(y-2)^2}$

$= \pm\sqrt{4y-y^2}$

or

$r = 4 \sin\theta$;



$y = 2 \rightarrow$

$r \sin\theta = 2 \rightarrow$

$r = 2\csc\theta$

c.) polar coordinates.

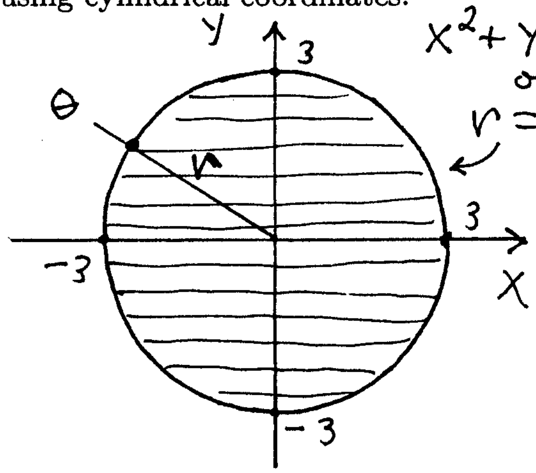
$$\begin{cases} \frac{\pi}{4} \le \theta \le \frac{3}{4}\pi, \\ 2\csc\theta \le r \le 4\sin\theta \end{cases}$$

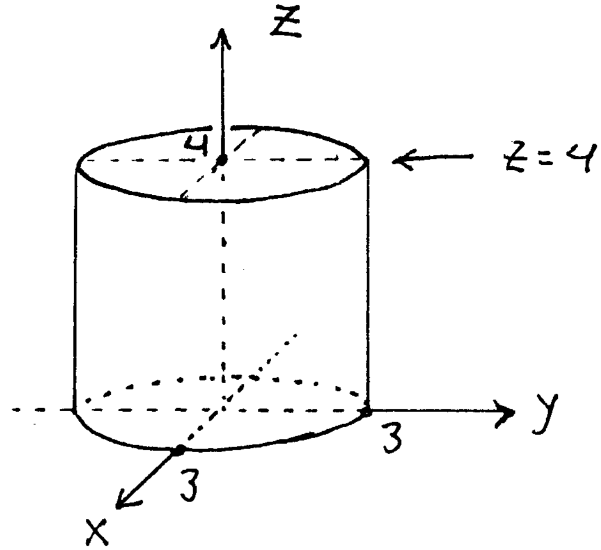
2.) Consider the solid region R inside the cylinder $x^2 + y^2 = 9$ which is bounded above by the plane $z = 4$ and below by the plane $z = 0$. Sketch the region and describe R using cylindrical coordinates.

$x^2 + y^2 = 9$

or

$r = 3$





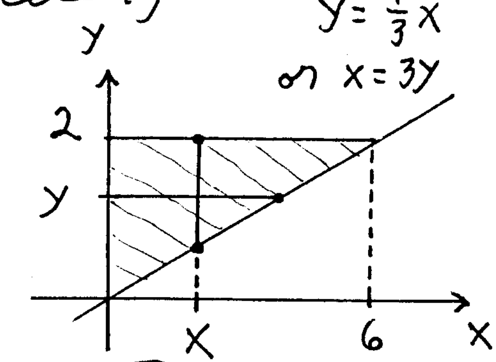
$$\begin{cases} 0 \le \theta \le 2\pi \\ 0 \le r \le 3 \\ 0 \le z \le 4 \end{cases}$$

3.) Evaluate each of the following double integrals.

a.) $\int_0^3 \int_0^2 xy^2\, dx\, dy = \int_0^3 \left(\frac{x^2}{2} y^2 \Big|_{x=0}^{x=2} \right) dy$

$= \int_0^3 2y^2\, dy = \frac{2}{3} y^3 \Big|_0^3 = 18.$

b.) $\int_0^6 \int_{(1/3)x}^2 \sin(y^2)\, dy\, dx$ (switch order)



$= \int_0^2 \int_0^{3y} \sin y^2\, dx\, dy$

$= \int_0^2 x \sin y^2 \Big|_{x=0}^{x=3y}\, dy$

$= \int_0^2 3y \sin y^2\, dy$

$= -\frac{3}{2} \cos y^2 \Big|_0^2 = -\frac{3}{2} \cos 4 + \frac{3}{2} \cos 0 = \frac{3}{2} - \frac{3}{2} \cos 4$

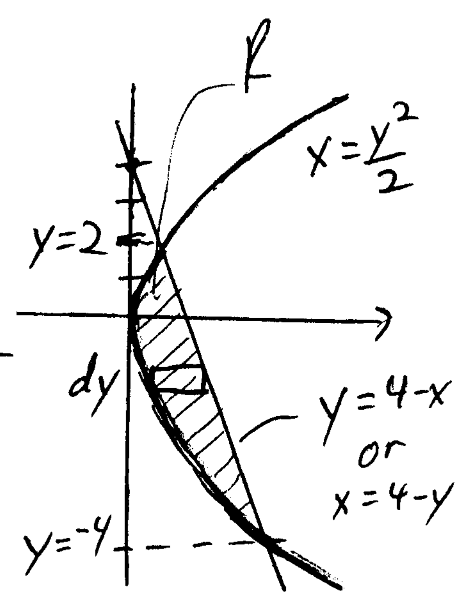
4.) Consider the flat region R bounded by the graphs of $y^2 = 2x$ and $x + y = 4$. Assume the density at the point $P = (x, y)$ is given by $\delta(x, y) = x^2 + y^2$. SET UP BUT DO NOT EVALUATE the double integral which represent the *mass* of R.

Intersection Pts:

$\frac{y^2}{2} = 4 - y \Rightarrow y^2 = 8 - 2y$

$\Rightarrow y^2 + 2y - 8 = 0$

$\Rightarrow (y-2)(y+4) = 0$



$dy: -4 \le y \le 2$

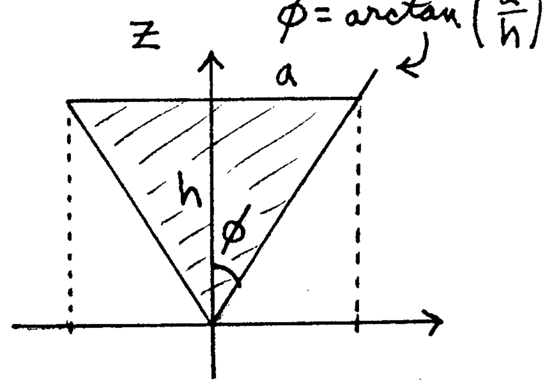
$\frac{y^2}{2} \le x \le 4 - y$ & $\delta(x,y) = x^2 + y^2$

$\Rightarrow m = \int_{-4}^{2} \int_{\frac{y^2}{2}}^{4-y} x^2 + y^2\, dx\, dy$

6.) Set up and EVALUATE a triple integral using spherical coordinates representing the *volume* of a right circular cone of radius a and height h.

$z = h \rightarrow \rho \cos\phi = h$

$\rightarrow \rho = h \sec\phi \, ;$



$$\mathrm{Vol} = \int_0^{2\pi} \int_0^{\arctan(a/h)} \int_0^{h\sec\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= \int_0^{2\pi} \int_0^{\arctan(a/h)} \left(\frac{\rho^3}{3} \cdot \sin\phi \, \Big|_{\rho=0}^{\rho=h\sec\phi} \right) d\phi \, d\theta$$

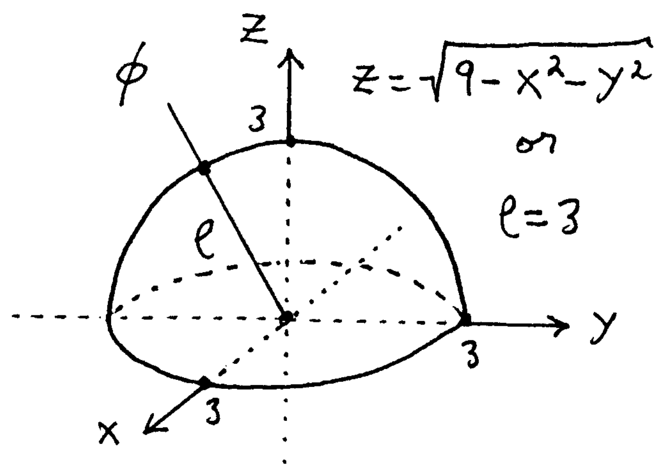
$$= \int_0^{2\pi} \int_0^{\arctan\left(\frac{a}{h}\right)} \frac{h^3}{3} \sec^3\phi \sin\phi \, d\phi \, d\theta$$

$$= \int_0^{2\pi} \int_0^{\arctan\left(\frac{a}{h}\right)} \frac{h^3}{3} \cdot \sec^2\phi \tan\phi \, d\phi \, d\theta$$

$$= \int_0^{2\pi} \frac{h^3}{3} \cdot \frac{1}{2} \tan^2\phi \, \Big|_{\phi=0}^{\phi=\arctan\left(\frac{a}{h}\right)} d\theta$$

$$= \int_0^{2\pi} \left(\frac{h^3}{6} \cdot \frac{a^2}{h^2} \right) d\theta = \frac{1}{6} a^2 h \, \theta \Big|_0^{2\pi} = \frac{1}{3} \pi a^2 h$$

7.) Consider the solid region R enclosed by the hemisphere $z = \sqrt{9 - x^2 - y^2}$. SET UP BUT DO NOT EVALUATE triple integrals in spherical coordinates which represent the *average value* of function $f(x, y, z) = x + z$ over region R.



$$\text{Vol. of } R = \int_R 1 \, dV$$

$$= \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \int_0^3 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta \, ;$$

$$\text{AVE} = \frac{1}{\text{Vol. } R} \int_R f(P) \, dV$$

$$= \frac{1}{\text{Vol. } R} \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \int_0^3 (\rho \cos\theta \sin\phi + \rho \cos\phi) \cdot \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

The following EXTRA CREDIT PROBLEM is worth . This problem is OPTIONAL.

1.) Consider the solid region R bounded below by the plane $z = 0$, on the sides by the cylinder $(x-1)^2 + y^2 = 1$, and on the top by the cone $z = \sqrt{x^2+y^2}$. SET UP BUT DO NOT EVALUATE a triple in spherical corrdinates, which represents the volume of R.

$z = \sqrt{x^2+y^2}$ (cone) $\rightarrow$

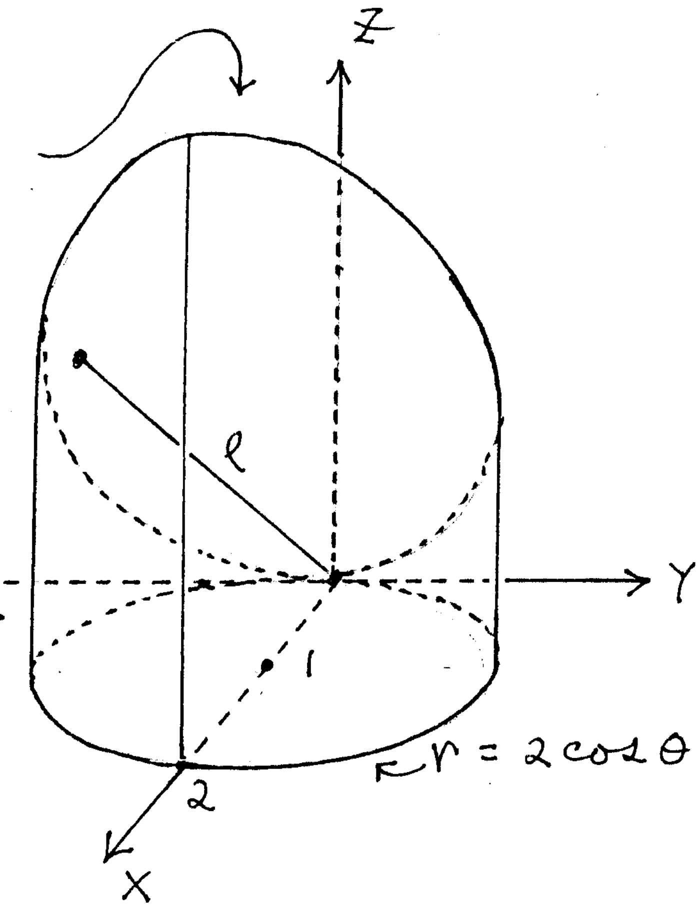
$\phi = \frac{\pi}{4}$

$(x-1)^2 + y^2 = 1 \rightarrow$

$r = 2\cos\theta \rightarrow$

$\rho \sin\phi = 2\cos\theta \rightarrow$

$\rho = \dfrac{2\cos\theta}{\sin\phi}$



$$\text{Vol} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_{0}^{\frac{2\cos\theta}{\sin\phi}} 1 \cdot \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$