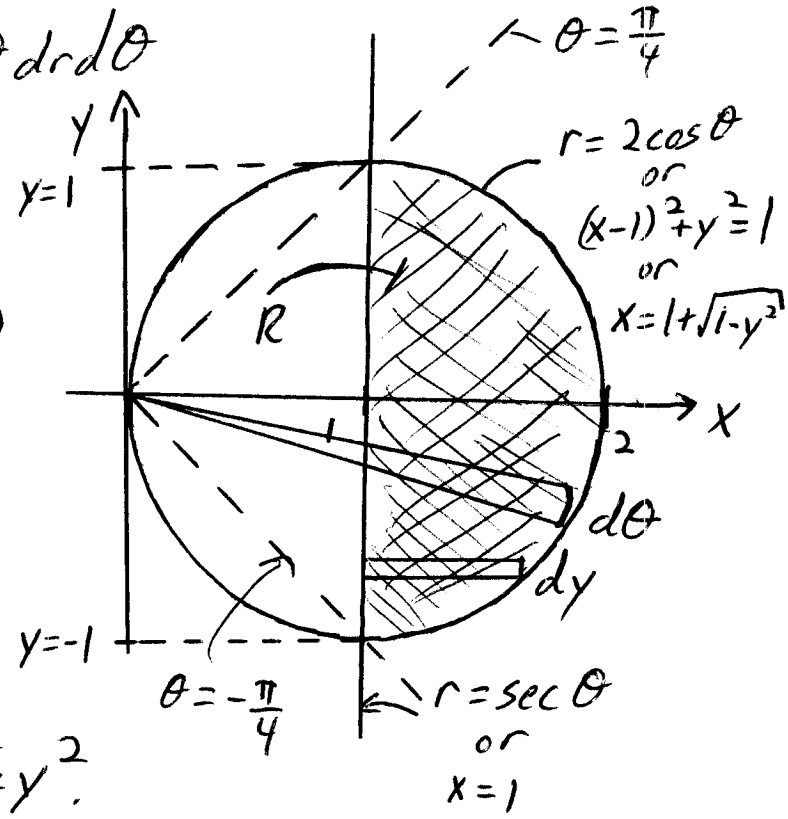
Math 21D
Vogler

Converting Polar to Rectangular Ex

Ex Convert the following into a rectangular double integral.

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \int_{\sec\theta}^{2\cos\theta} r^3 \sin^2\theta \, dr\, d\theta$$



The polar bounds for R are

$$\left.\begin{array}{l} d\theta: -\pi/4 \le \theta \le \pi/4 \\ \sec\theta \le r \le 2\cos\theta \end{array}\right\} \Longrightarrow \text{graph}$$

Then, the rectangular bounds are

$$dy: -1 \le y \le 1$$
$$1 \le x \le 1 + \sqrt{1-y^2}$$

Notice that $r^2\sin^2\theta = (r\sin\theta)^2 = y^2$.

Therefore, we have

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \int_{\sec\theta}^{2\cos\theta} r^3 \sin^2\theta \, dr\, d\theta = \iint_R r^2 \sin^2\theta \, dA \quad (\text{Note: } r\,dr\,d\theta = dA)$$

$$= \int_{-1}^{1} \int_{1}^{1+\sqrt{1-y^2}} y^2 \, dx\, dy.$$