Math 21D
Vogler

Divergence Theorem

Defn Let $\vec{F}(x,y,z) = M(x,y,z)\vec{i} + N(x,y,z)\vec{j} + P(x,y,z)\vec{k}$ be a vector field defined on solid D. The divergence of $\vec{F}$ at (x,y,z) is the scalar function

$$\text{div } \vec{F} = \vec{\nabla}\cdot\vec{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z} = M_x + N_y + P_z$$

Note: Divergence can be shown to be a measure of expansion $(\text{div } \vec{F} > 0)$ or compression $(\text{div } \vec{F} < 0)$ of a fluid or gas at a point (x,y,z).

Divergence Theorem

Let $\vec{F}(x,y,z) = M(x,y,z)\vec{i} + N(x,y,z)\vec{j} + P(x,y,z)\vec{k}$ be a vector field defined on a solid D, enclosed by surface S with outward-pointing unit normal vector $\vec{n}$

$$\Rightarrow \underbrace{\iint_S \vec{F}\cdot\vec{n}\, d\sigma}_{\substack{\text{Outward Flux} \\ \text{across Boundary}}} = \underbrace{\iiint_D \overbrace{\vec{\nabla}\cdot\vec{F}}^{\text{div } \vec{F}}\, dV}_{\substack{\text{Divergence Integral} \\ \text{or} \\ \text{Expansion Rates} \\ \text{across Interior}}}$$