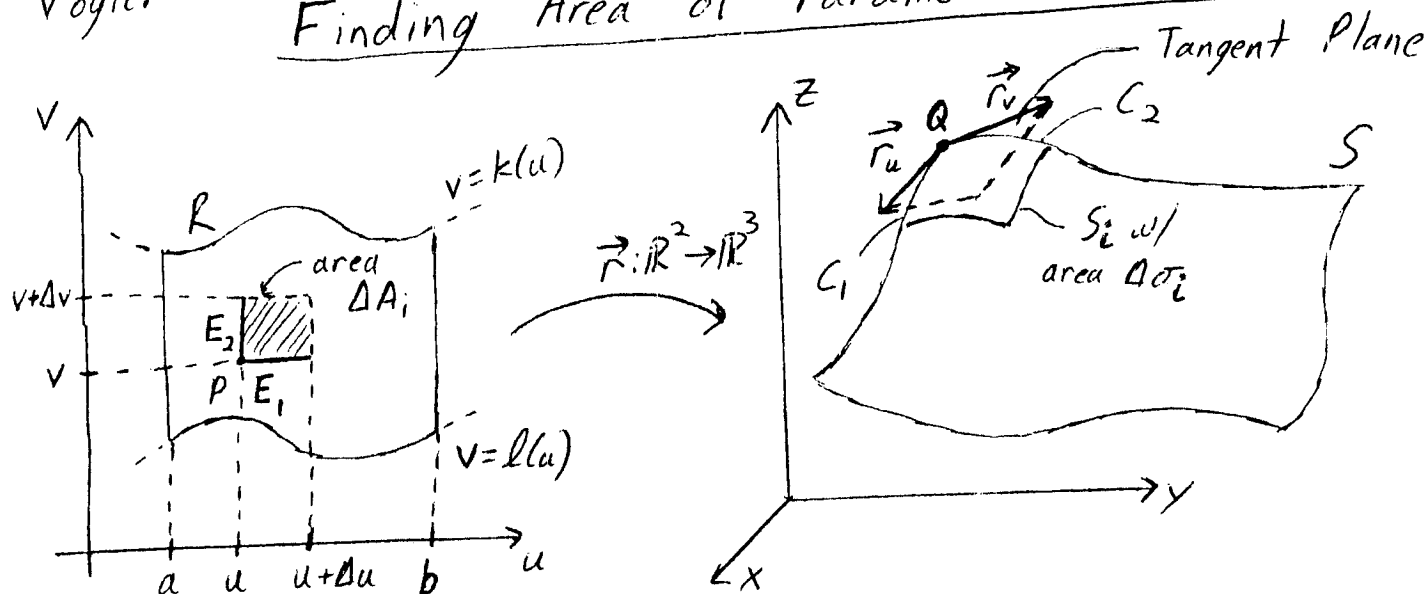


Finding Area of Parametrized Surface



Let S be surface parametrized by $\vec{r}: \mathbb{R}^2 \rightarrow \mathbb{R}^3$
 $\vec{r}(u,v) = f(u,v)\hat{i} + g(u,v)\hat{j} + h(u,v)\hat{k}$ for (u,v)
 in $R: a \leq u \leq b, l(u) \leq v \leq k(u)$.

Assume $\vec{r}(P) = Q$; let path C_1/C_2 be image of edge E_1/E_2 under mapping $\vec{r}$.

(Recall: $y = f(x) \Rightarrow \Delta f \approx f'(x) \Delta x$)

$\Rightarrow$ path $C_1 \approx \Delta \vec{r} \text{ (along } E_1) \approx \vec{r}_u(Q) \Delta u$

& path $C_2 \approx \Delta \vec{r} \text{ (along } E_2) \approx \vec{r}_v(Q) \Delta v$

Recall: Area of $\square$ formed by vectors $\vec{r}_u \Delta u$ & $\vec{r}_v \Delta v$ is (magnitude of cross product)

$$|\vec{r}_u \Delta u \times \vec{r}_v \Delta v| = |\vec{r}_u \times \vec{r}_v| \underbrace{\Delta v \cdot \Delta u}_{\Delta A_i}$$

$\Rightarrow$ area $\Delta \sigma_i \approx \underbrace{|\vec{r}_u \times \vec{r}_v|}_{\text{'Exchange Rate'}} \Delta A_i$ so that...

Defn If surface S is given by $\vec{r}(u,v): \mathbb{R}^2 \rightarrow \mathbb{R}^3$
 $\vec{r}(u,v) = f(u,v)\hat{i} + g(u,v)\hat{j} + h(u,v)\hat{k}$ for (u,v) in R ,
 where $R: a \leq u \leq b, l(u) \leq v \leq k(u)$
 $\Rightarrow$ Surface Area $S = \iint_R |\vec{r}_u \times \vec{r}_v| dA$

For Implicit Surfaces: (See book for derivations)

- Let $F(x,y,z)=c$ be implicit surface above bounded plane region R

$$\Rightarrow \text{Surface Area} = \iint_R \frac{|\vec{\nabla} F|}{|\vec{\nabla} F \cdot \vec{p}|} dA,$$

where $\vec{p} = \hat{i}, \hat{j}, \text{ or } \hat{k}$ is normal to R & $\vec{\nabla} F \cdot \vec{p} \neq 0$

- Let $z = f(x,y)$ be over region R in xy -plane

$$\Rightarrow \text{Surface Area} = \iint_R \sqrt{f_x^2 + f_y^2 + 1} dA$$