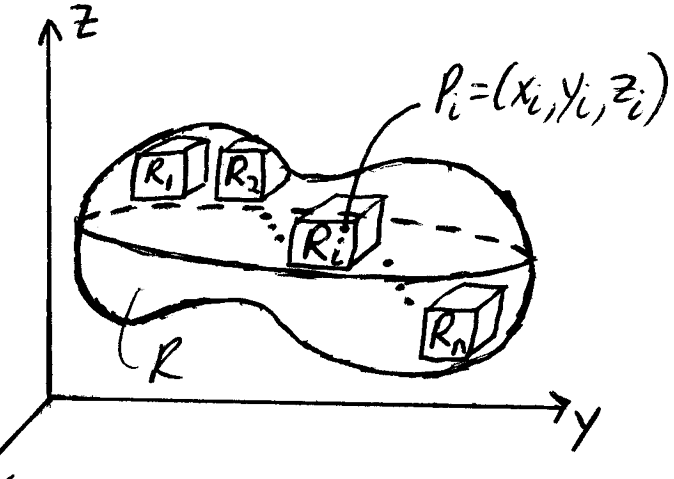
Math 21D

Vogler

Triple Integrals



Defn

- Let function $f(P)$ be defined on region R.
- Let $R_1, R_2, \ldots, R_n$ be subdivision of R with corresponding volumes $\Delta V_1, \Delta V_2, \ldots, \Delta V_n$.
- Let $P_i = (x_i, y_i, z_i)$ be arbitrary point in R_i for all i.

Then the <u>definite</u> <u>triple</u> <u>integral</u> of $f(P)$ over R is

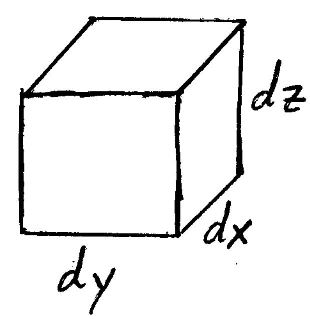
$$\iiint_R f(P)\, dV := \lim_{n\to\infty} \sum_{i=1}^{n} f(P_i)\Delta V_i$$

Notes: 1) Define the <u>mesh</u> as $\Delta V := \max_{1\le i\le n} \{\Delta V_i\}$

2) $n \to \infty$ is equivalent to $\Delta V \to 0$.

3) dV can be thought of an infinitely small 3D "lego" piece.

4) $dV = dx\,dy\,dz = dx\,dz\,dy = dy\,dx\,dz = dy\,dz\,dx = dz\,dx\,dy = dz\,dy\,dx$ are called <u>rectangular</u> <u>coordinates</u>.



<u>Facts</u>: 1) $\iiint_R 1\, dV = \text{Volume of } R =: V_R$

2) Average value of $f(P)$ over solid region R is

$$\text{Ave} := \frac{1}{V_R}\iiint_R f(P)\, dV = \frac{1}{\iiint_R 1\, dV}\iiint_R f(P)\, dV$$