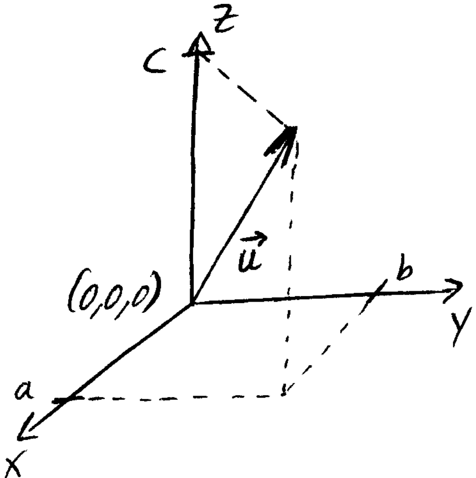
Math 21D
Vogler

Vector Functions Review

I) Vector $\vec{u} = a\vec{i} + b\vec{j} + c\vec{k}$ in 3D-space is a quantity with direction & magnitude (i.e. length), where $\vec{i} = (1,0,0)$, $\vec{j} = (0,1,0)$, & $\vec{k} = (0,0,1)$

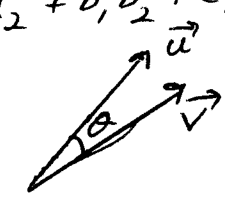


a) Its magnitude is
$$|\vec{u}| = \sqrt{a^2+b^2+c^2}$$

b) Its direction is the unit vector (length = 1)
$$\frac{\vec{u}}{|\vec{u}|} = \frac{a}{\sqrt{a^2+b^2+c^2}}\vec{i} + \frac{b}{\sqrt{a^2+b^2+c^2}}\vec{j} + \frac{c}{\sqrt{a^2+b^2+c^2}}\vec{k}$$

Note: For vectors in 2D-space assume $c = 0$.

II) If $\vec{u} = a_1\vec{i} + b_1\vec{j} + c_1\vec{k}$ & $\vec{v} = a_2\vec{i} + b_2\vec{j} + c_2\vec{k}$
$\Rightarrow$ the dot product b/w $\vec{u}$ & $\vec{v}$ is $\vec{u}\cdot\vec{v} = a_1a_2 + b_1b_2 + c_1c_2$



a) The angle θ b/w $\vec{u}$ & $\vec{v}$ is given by
$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}$$

b) $\vec{u} \perp \vec{v} \iff \vec{u}\cdot\vec{v} = 0$

c) $|\vec{u}|^2 = a^2+b^2+c^2 = \vec{u}\cdot\vec{u}$

III) Line L passing through point (a,b,c) in direction of vector $\vec{u} = u_1\vec{i} + v_1\vec{j} + w_1\vec{k}$ is given parametrically by

$$L: \begin{cases} x = a + u_1 t \\ y = b + v_1 t \\ z = c + w_1 t \end{cases} \quad \text{for } -\infty < t < \infty$$

Defn The <u>vector function</u> $\vec{r}(t) = f(t)\vec{i} + g(t)\vec{j} + h(t)\vec{k}$ $(r: \mathbb{R} \to \mathbb{R}^3)$ assigns a vector in 3D-space to each real # t (i.e. time) in its domain.

Defn The <u>derivative</u> <u>of</u> <u>a vector</u> <u>function</u> $\vec{r}(t) = f(t)\vec{i} + g(t)\vec{j} + h(t)\vec{k}$ is $\frac{d}{dt}\vec{r}(t) = \vec{r}\,'(t) := \lim_{h \to 0} \frac{\vec{r}(t+h) - \vec{r}(t)}{h} = f'(t)\vec{i} + g'(t)\vec{j} + h'(t)\vec{k}$

<u>Motion in 3D using vector functions</u>

I) Position vector: $\vec{r}(t) = f(t)\vec{i} + g(t)\vec{j} + h(t)\vec{k}$

II) Velocity vector: $\vec{v}(t) = \vec{r}\,'(t) = f'(t)\vec{i} + g'(t)\vec{j} + h'(t)\vec{k}$

a) $\vec{v}(t)$ points in direction of motion.

b) $\vec{v}(t)$ is tangent to path traced out by $\vec{r}(t)$

c) $|\vec{v}(t)|$ is speed of motion.

III) Acceleration vector: $\vec{a}(t) = \vec{v}\,'(t) = \vec{r}\,''(t) = f''(t)\vec{i} + g''(t)\vec{j} + h''(t)\vec{k}$

<u>Differentiation Rules</u>

Refer to Section 13.1 pg. 730

Defn The <u>definite integral</u> of $r(t) = f(t)\vec{i} + g(t)\vec{j} + h(t)\vec{k}$ is

$$\int_a^b \vec{r}(t)\,dt = \left(\int_a^b f(t)\,dt\right)\vec{i} + \left(\int_a^b g(t)\,dt\right)\vec{j} + \left(\int_a^b h(t)\,dt\right)\vec{k}$$

IV) If $\vec{u} = a_1\vec{i} + b_1\vec{j} + c_1\vec{k}$ & $\vec{v} = a_2\vec{i} + b_2\vec{j} + c_2\vec{k}$

$\Rightarrow$ the cross product b/w $\vec{u}$ & $\vec{v}$ is

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = (b_1c_2 - b_2c_1)\vec{i} - (a_1c_2 - a_2c_1)\vec{j} + (a_1b_2 - a_2b_1)\vec{k}$$