Section 13.2

1.) $\int_0^1 [t^3 \vec{i} + 7\vec{j} + (t+1)\vec{k}]\,dt$

$= \left(\frac{1}{4}t^4\Big|_0^1\right)\vec{i} + \left(7t\Big|_0^1\right)\vec{j} + \left(\frac{1}{2}t^2+t\right)\Big|_0^1 \cdot \vec{k}$

$= \frac{1}{4}\vec{i} + 7\vec{j} + \frac{3}{2}\vec{k}$

4.) $\int_0^{\frac{\pi}{3}} [(\sec t \tan t)\vec{i} + (\tan t)\vec{j} + (2\cos t \sin t)\vec{k}]\,dt$

$= \left(\sec t\Big|_0^{\frac{\pi}{3}}\right)\vec{i} + \left(\ln|\sec t|\Big|_0^{\frac{\pi}{3}}\right)\vec{j} + \left(\sin^2 t\Big|_0^{\frac{\pi}{3}}\right)\vec{k}$

$= (\sec\frac{\pi}{3} - \sec 0)\vec{i} + (\ln|\sec\frac{\pi}{3}| - \ln|\sec 0|)\vec{j}$

$+ (\sin^2\frac{\pi}{3} - \sin^2 0)\vec{k}$

$= (2-1)\vec{i} + (\ln 2 - \cancel{\ln 1}^{0})\vec{j} + (\frac{3}{4} - 0)\vec{k}$

$= 1\cdot\vec{i} + \ln 2\cdot\vec{j} + \frac{3}{4}\vec{k}$

6.) $\int_0^1 \left[\frac{2}{\sqrt{1-t^2}}\vec{i} + \frac{\sqrt{3}}{1+t^2}\vec{k}\right]dt$

$= \left(2\arcsin t\Big|_0^1\right)\vec{i} + \left(\sqrt{3}\arctan t\Big|_0^1\right)\vec{k}$

$= (2\arcsin 1 - 2\arcsin 0)\vec{i}$

$+ (\sqrt{3}\arctan 1 - \sqrt{3}\arctan 0)\vec{k}$

$= (2\cdot\frac{\pi}{2} - 2\cdot 0)\vec{i} + (\sqrt{3}\cdot\frac{\pi}{4} - \sqrt{3}\cdot 0)\vec{k}$

$= (\pi)\vec{i} + \left(\frac{\sqrt{3}}{4}\pi\right)\vec{k}$

15.) $\vec{r}\,''(t) = -32\vec{k} \longrightarrow$

$\vec{r}\,'(t) = c_1\vec{i} + c_2\vec{j} + (-32t + c_3)\vec{k}$

and $\vec{r}\,'(0) = 8\vec{i} + 8\vec{j} + 0\cdot\vec{k}$

$c_1 = 8,\ c_2 = 8,\ -32(0) + c_3 = 0 \rightarrow c_3 = 0 \longrightarrow$

$\vec{r}\,'(t) = 8\vec{i} + 8\vec{j} + (-32t)\vec{k} \longrightarrow$

$\vec{r}(t) = (8t + c_1)\vec{i} + (8t + c_2)\vec{j} + (-16t^2 + c_3)\vec{k}$

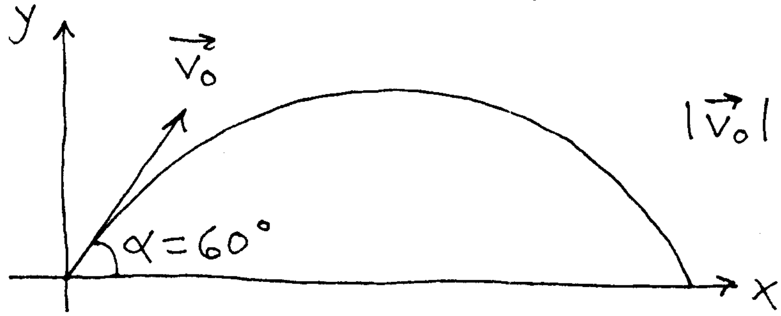
and $\vec{r}(0) = 0 \cdot \vec{i} + 0 \cdot \vec{j} + 100\,\vec{k} \longrightarrow$

$8(0) + C_1 = 0,\ 8(0) + C_2 = 0,\ -16(0)^2 + C_3 = 100 \longrightarrow$

$C_1 = 0,\ C_2 = 0,$ and $C_3 = 100 \longrightarrow$

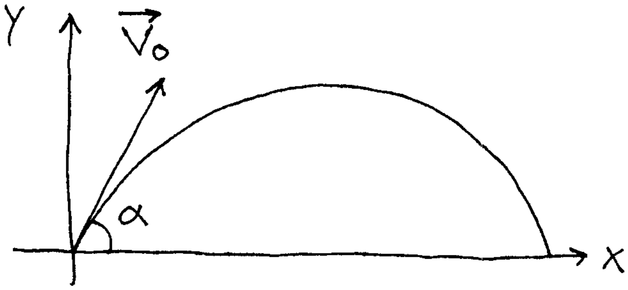
$$\vec{r}(t) = (8t)\,\vec{i} + (8t)\,\vec{j} + (100 - 16t^2)\,\vec{k}$$

$x(t) = |\vec{v}_0| \cos\alpha \cdot t \,,\; y(t) = |\vec{v}_0| \sin\alpha \cdot t - \frac{1}{2} g \cdot t^2$

19.) 

$|\vec{v}_0| = 840$ m./sec.

$x(t) = (840)(\cos 60°) t = (840)(\frac{1}{2}) t = 420 t \rightarrow$
$x(t) = 420 t$; if $x(t) = 21$ km $= 21{,}000$ m. $\rightarrow$
$420 t = 21{,}000 \rightarrow \quad t = 50$ sec.

20.) 

$y(t) = |\vec{v}_0| \sin\alpha \cdot t - \frac{1}{2}(9.8) t^2$
$= |\vec{v}_0| \sin\alpha \cdot t - 4.9 t^2$; find flight time $\rightarrow \quad y(t) = 0 \rightarrow \quad t\,(|\vec{v}_0| \sin\alpha - 4.9 t) = 0$
$\rightarrow \quad t = \dfrac{|v_0| \sin\alpha}{4.9}$ sec. ; downrange distance is

$x(t) = |\vec{v}_0| \cos\alpha \cdot t = |\vec{v}_0| \cos\alpha \cdot \dfrac{|\vec{v}_0| \sin\alpha}{4.9} \rightarrow$

$x = \dfrac{|\vec{v}_0|^2}{4.9} \sin\alpha \cos\alpha$; find angle α which maximizes x :

$$\frac{dx}{d\alpha} = \frac{|\vec{V}_0|^2}{4.9}\left(\sin\alpha\cdot(-\sin\alpha) + \cos\alpha\cdot\cos\alpha\right)$$

$$= \frac{|\vec{V}_0|^2}{4.9}\left(\cos^2\alpha - \sin^2\alpha\right) = 0 \rightarrow$$

$\cos^2\alpha = \sin^2\alpha \rightarrow \alpha = 45°$, then

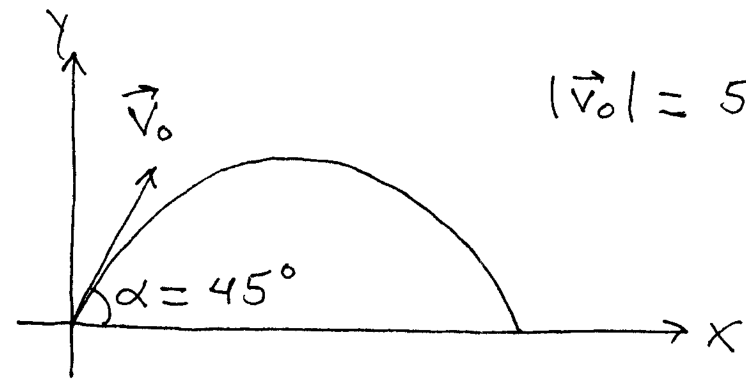
$$X = \frac{|\vec{V}_0|^2}{4.9}\sin 45°\cdot\cos 45° = \frac{|\vec{V}_0|}{4.9}\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \rightarrow$$

$X = \dfrac{|\vec{V}_0|^2}{9.8}$; if max. range is 24,500 m then $\dfrac{|\vec{V}_0|^2}{9.8} = 24,500 \rightarrow$

$$|\vec{V}_0|^2 = (24,500)(9.8) \longrightarrow$$

$$|\vec{V}_0| = \sqrt{240,100} = 490 \text{ m./sec.}$$

21.)



$|\vec{V}_0| = 500$ m./sec.,

$$x(t) = (500)\cos 45°\cdot t = (500)\frac{\sqrt{2}}{2}t = 250\sqrt{2}\,t \rightarrow$$

$\underline{x(t) = 250\sqrt{2}\,t}$;

$$y(t) = (500)\sin 45°\cdot t - \frac{1}{2}(9.8)t^2$$

$$= (500)\left(\frac{\sqrt{2}}{2}\right)t - 4.9t^2 = 250\sqrt{2}\,t - 4.9t^2 \rightarrow$$

$\underline{y(t) = 250\sqrt{2}\,t - 4.9t^2}$;

a.) find flight time : $y(t) = 0 \rightarrow$

$t(250\sqrt{2} - 4.9t) = 0 \rightarrow t = \frac{250\sqrt{2}}{4.9} \approx 72.15$ sec.;

distance downrange:

$x(72.15) = 250\sqrt{2} \cdot (72.15) \approx 25{,}510.2$ m.

b.) If $x(t) = 5000$ m $\rightarrow 250\sqrt{2}\, t = 5000 \rightarrow$

$t = \dfrac{5000}{250\sqrt{2}} \approx 14.14$ sec., and height is

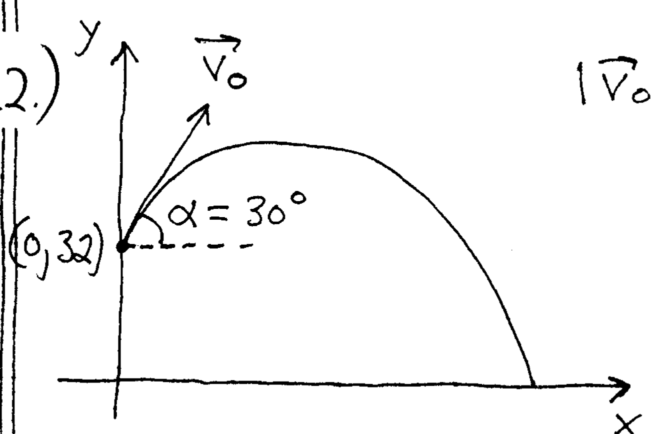
$y(14.14) = 250\sqrt{2}\,(14.14) - 4.9\,(14.14)^2 \approx 4019.5$ m.

c.) maximum $y(t)$: $y'(t) = 0 \longrightarrow$

$250\sqrt{2} - 9.8t = 0 \rightarrow t = \dfrac{250\sqrt{2}}{9.8} \approx 36.1$ sec.

$\rightarrow y(36.1) = 250\sqrt{2}\,(36.1) - 4.9\,(36.1)^2 \approx 6377.5$ m.

22.)



$|\vec{v}_0| = 32$ ft./sec.

$x(t) = (32) \cdot \cos 30° \cdot t$ and

$y(t) = b + |\vec{v}_0| \sin\alpha \cdot t - \frac{1}{2} g t^2$

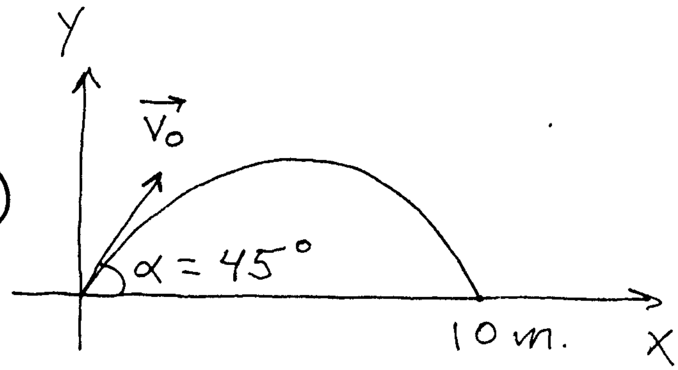
$= 32 + (32) \sin 30° \cdot t - \frac{1}{2}(32) t^2$

$= 32 + (32)(\frac{1}{2}) t - 16 t^2$

$\to$ $y(t) = 32 + 16t - 16t^2$; flight time $\to$
$y(t) = 0 \to 32 + 16t - 16t^2 = 0 \longrightarrow$
$-16(t^2 - t - 2) = -16(t-2)(t+1) = 0 \to$
$\boxed{t = 2 \text{ sec.}}$; distance downrange is

$x(2) = (32) \cdot \cos 30° \cdot (2) = (32)\left(\frac{\sqrt{3}}{2}\right)(2)$
$= 32\sqrt{3} \approx \boxed{55.4 \text{ ft}}$.

23.)



a.) Downrange distance:

$$x(t) = \frac{2|\vec{V}_0|^2 \cos\alpha \sin\alpha}{g} \rightarrow$$

$$10 = \frac{2|\vec{V}_0|^2 \cos 45° \sin 45°}{9.8} \rightarrow$$

$$10 = \frac{|\vec{V}_0|^2 (\sqrt{2}/2)(\sqrt{2}/2)}{4.9} \rightarrow$$

$$49 = |\vec{V}_0|^2 \left(\tfrac{1}{2}\right) \rightarrow 98 = |\vec{V}_0|^2 \rightarrow$$

$$|\vec{V}_0| \approx 9.9 \text{ m./sec.}$$

b.) assume that $|\vec{V}_0| = 9.9$ m./sec. and downrange $x(t) = 6$ m.; then

$$x = \frac{2|\vec{V}_0|^2 \cos\alpha \sin\alpha}{g} \rightarrow$$

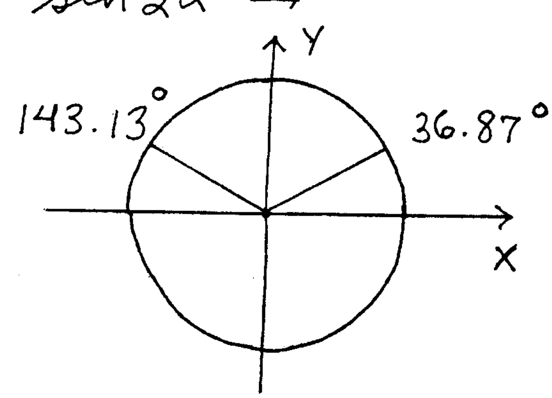
$$6 = \frac{2(9.9)^2}{9.8} \cos\alpha \sin\alpha \rightarrow$$

$$\frac{(6)(9.8)}{(9.9)^2} = 2\sin\alpha\cos\alpha = \sin 2\alpha \rightarrow$$

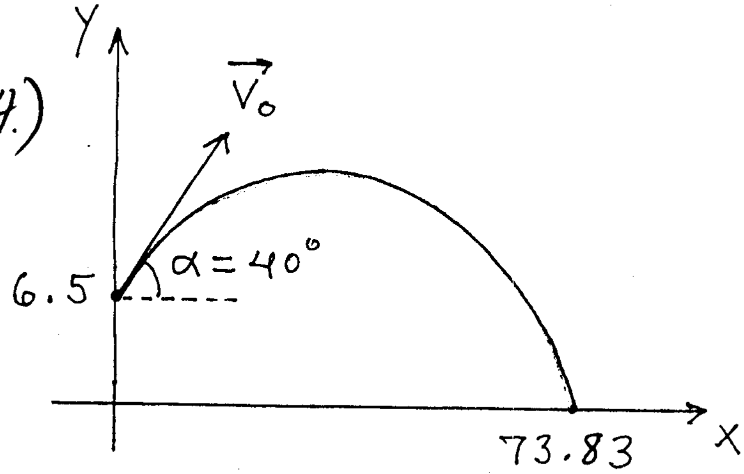
$\sin 2\alpha \approx 0.6 \rightarrow$

$2\alpha \approx 36.87$ or $143.13 \rightarrow$

$\alpha \approx 18.4°$ or $\alpha \approx 71.6°$



34.)



$$\begin{cases} x(t) = |\vec{V}_0| \cdot \cos 40° \cdot t \\ y(t) = 6.5 + |\vec{V}_0| \sin 40° t - \frac{1}{2}(32)t^2 \end{cases}$$

$$\begin{cases} 73.83 = |\vec{V}_0| \cos 40° \cdot t \\ 0 = 6.5 + |\vec{V}_0| \sin 40° t - 16t^2 \end{cases} \rightarrow$$

$$t = \frac{73.83}{|\vec{V}_0| \cos 40°}$$ so that

$$0 = 6.5 + \cancel{|\vec{V}_0|} \sin 40° \cdot \left(\frac{73.83}{\cancel{|\vec{V}_0|} \cos 40°}\right) - 16\left(\frac{73.83}{|\vec{V}_0| \cos 40°}\right)^2$$

$$\rightarrow 16 \frac{(73.83)^2}{|\vec{V}_0|^2 \cos^2 40°} = 6.5 + \frac{73.83 \sin 40°}{\cos 40°} \rightarrow$$

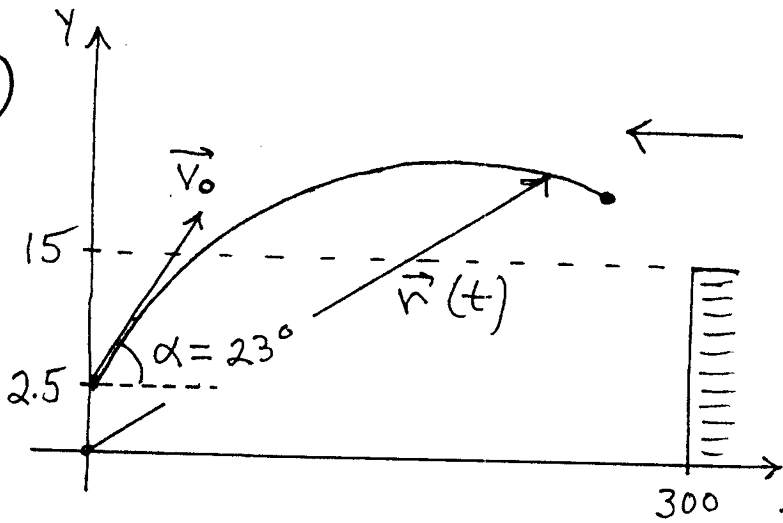
$$|\vec{V}_0|^2 = \frac{\frac{16(73.83)^2}{\cos^2 40°}}{6.5 + \frac{73.83 \sin 40°}{\cos 40°}} \cdot \frac{\cos^2 40°}{\cos^2 40°} \rightarrow$$

$$|\vec{V}_0|^2 = \frac{16(73.83)^2}{6.5 \cos^2 40° + 73.83 \sin 40° \cos 40°}$$

$$\approx 2171.2$$ so

$$|\vec{V}_0| \approx \sqrt{2171.2} \approx 46.6 \text{ ft./sec.}$$

36.)



$-14\vec{i}$ mph when $t=0$

$|\vec{v}_0| = 145$ ft./sec.

$x(t) = (|\vec{v}_0| \cos\alpha - 14)t = (145 \cos 23° - 14)t$,

$y(t) = 2.5 + |\vec{v}_0| \sin\alpha \cdot t - \frac{1}{2}gt^2$

$= 2.5 + (145)\sin 23° \cdot t - \frac{1}{2}(32)t^2$

$= 2.5 + (145)\sin 23° \cdot t - 16t^2$ so

a.) $\vec{r}(t) = (45\cos 23° - 14)t \cdot \vec{i}$

$+ (2.5 + (145)\sin 23° \cdot t - 16t^2) \cdot \vec{j}$

b.) at max. height $y'(t) = 0$:

$y'(t) = (145)\sin 23° - 32t = 0 \longrightarrow$

$t = \frac{(145)\sin 23°}{32} \approx \boxed{1.771 \text{ sec.}}$ and

$y = 2.5 + (145)\sin 23° (1.771) - 16(1.771)^2 \longrightarrow$

$y_{max} \approx \boxed{52.65 \text{ ft.}}$

c.) Hit ground : $y(t) = 0 \rightarrow$

$2.5 + (145)\sin 23° \cdot t - 16t^2 = 0 \rightarrow$

$$t = \frac{-145\sin 23° \pm \sqrt{145^2 \sin^2 23° - 4(2.5)(-16)}}{2(-16)}$$

$$\approx \frac{-56.656 \pm \sqrt{3369.9}}{-32}$$

$$= \frac{-56.656 \pm 58.051}{-32} \approx \boxed{3.585 \text{ sec.}}$$;

downrange dist.

$$x = (145\cos 23° - 14)(3.585) \approx \boxed{428.31 \text{ ft.}}$$

d.) If $y(t) = 20$ ft., then

$2.5 + (145)\sin 23° \cdot t - 16t^2 = 20 \rightarrow$

$0 = 16t^2 + {}^{-}145\sin 23° \cdot t + 17.5 \rightarrow$

$$t = \frac{145\sin 23° \pm \sqrt{(-145\sin 23°)^2 - 4(16)(17.5)}}{2(16)}$$

$$\approx \frac{56.656 \pm \sqrt{2089.904}}{32}$$

$$\approx 3.2 \text{ sec. or } 0.342 \text{ sec.}$$;

$x(3.2) = (145\cos 23° - 14)(3.2) \approx 382.3$ ft.

$x(0.342) = (145\cos 23° - 14)(0.342) \approx 40.86$ ft.,

e.) Yes, since fence is 15 ft. high and 300 ft. from the batter.