

Section 16.1

1.) $\vec{r}(t) = t\vec{i} + (1-t)\vec{j} + 0\cdot\vec{k}, 0 \leq t \leq 1 \rightarrow$

$$\begin{cases} x=t \rightarrow (\text{SUB}) \\ y=1-t \rightarrow y=1-x \\ z=0 \end{cases} \quad (\text{c.})$$

2.) $\vec{r}(t) = 1\cdot\vec{i} + 1\cdot\vec{j} + t\cdot\vec{k}, -1 \leq t \leq 1 \rightarrow$

$$\begin{cases} x=1 \\ y=1 \\ z=t \end{cases} \quad (\text{e.})$$

3.) $\vec{r}(t) = (2\cos t)\vec{i} + (2\sin t)\vec{j} + 0\cdot\vec{k}, 0 \leq t \leq 2\pi$

$$\begin{cases} x=2\cos t \\ y=2\sin t \\ z=0 \end{cases} \rightarrow \begin{aligned} x^2 + y^2 &= (2\cos t)^2 + (2\sin t)^2 \\ &= 4\cos^2 t + 4\sin^2 t \\ &= 4(\cos^2 t + \sin^2 t) = 4 \cdot 1 = 2^2 \end{aligned}$$

(g.)

4.) $\vec{r}(t) = t\vec{i} + 0\cdot\vec{j} + 0\cdot\vec{k}, -1 \leq t \leq 1$

$$\begin{cases} x=t \\ y=0 \\ z=0 \end{cases} \quad (\text{a.})$$

5.) $\vec{r}(t) = t\vec{i} + t\vec{j} + t\vec{k}, 0 \leq t \leq 2$

$$\begin{cases} x=t \\ y=t \\ z=t \end{cases} \quad (\text{d.})$$

$$6.) \vec{r}(t) = 0 \cdot \vec{i} + t \cdot \vec{j} + (2-2t) \vec{k}, \quad 0 \leq t \leq 1 \rightarrow$$

$$\begin{cases} x=0 \\ y=t \\ z=2-2t \end{cases} \rightarrow \text{(sub)} \quad \text{(b.)}$$

$$z=2-2t \rightarrow z=2-2y$$

$$7.) \vec{r}(t) = 0 \cdot \vec{i} + (t^2-1) \vec{j} + 2t \cdot \vec{k}, \quad -1 \leq t \leq 1 \rightarrow$$

$$\begin{cases} x=0 \\ y=t^2-1 \\ z=2t \end{cases} \rightarrow \text{(sub)} \quad \text{(f.)}$$

$$y = \left(\frac{1}{2}z\right)^2 - 1$$

$$8.) \vec{r}(t) = (2\cos t) \vec{i} + (0) \vec{j} + (2\sin t) \vec{k}, \quad 0 \leq t \leq \pi \rightarrow$$

$$\begin{cases} x=2\cos t \\ y=0 \\ z=2\sin t \end{cases} \rightarrow \begin{aligned} x^2 + z^2 &= 4\cos^2 t + 4\sin^2 t \\ &= 4(\cos^2 t + \sin^2 t) = 4 \cdot 1 = 2^2 \end{aligned}$$

(h.)

$$10.) C: \vec{r}(t) = t \vec{i} + (1-t) \vec{j} + 1 \cdot \vec{k}, \quad 0 \leq t \leq 1 \rightarrow$$

$$\vec{v}(t) = 1 \cdot \vec{i} + (-1) \cdot \vec{j} + 0 \cdot \vec{k} \rightarrow$$

$$|\vec{v}(t)| = \sqrt{(1)^2 + (-1)^2 + (0)^2} = \sqrt{2} \quad \text{then}$$

$$\int_C (x-y+z-2) \, ds = \int_0^1 (t - (1-t) + 1 - 2) |\vec{v}(t)| \, dt$$

$$= \int_0^1 (2t-2) \sqrt{2} \, dt = \sqrt{2} (t^2 - 2t) \Big|_0^1$$

$$= \sqrt{2} (1-2) - \sqrt{2} (0) = -\sqrt{2}$$

$$11.) C: \vec{r}(t) = 2t \cdot \vec{i} + t \cdot \vec{j} + (2-2t) \vec{k}, \quad 0 \leq t \leq 1 \rightarrow$$

$$\vec{v}(t) = 2 \cdot \vec{i} + 1 \cdot \vec{j} + (-2) \cdot \vec{k} \rightarrow$$

$$|\vec{v}(t)| = \sqrt{2^2 + 1^2 + (-2)^2} = \sqrt{9} = 3 \text{ then}$$

$$\begin{aligned} \int_C (xy + y + z) \, ds &= \int_0^1 (2t \cdot t + t + (2-2t)) \cdot |\vec{v}(t)| \, dt \\ &= \int_0^1 (2t^2 + t + 2 - 2t)(3) \, dt \\ &= 3 \int_0^1 (2t^2 - t + 2) \, dt = 3 \left(\frac{2}{3} t^3 - \frac{1}{2} t^2 + 2t \right) \Big|_0^1 \\ &= 3 \left(\frac{2}{3} - \frac{1}{2} + 2 \right) = 3 \left(\frac{4}{6} - \frac{3}{6} + \frac{12}{6} \right) \\ &= 3 \cdot \left(\frac{13}{6} \right) = \frac{13}{2} \end{aligned}$$

$$12.) \quad \vec{r}(t) = (4\cos t)\vec{i} + (4\sin t)\vec{j} + (3t)\vec{k}$$

$$\text{for } -2\pi \leq t \leq 2\pi \quad \xrightarrow{D}$$

$$\vec{v}(t) = (-4\sin t)\vec{i} + (4\cos t)\vec{j} + (3)\vec{k} \text{ and}$$

$$|\vec{v}(t)| = \sqrt{(-4\sin t)^2 + (4\cos t)^2 + (3)^2}$$

$$= \sqrt{16(\underbrace{\sin^2 t + \cos^2 t}_1) + 9}$$

$$= \sqrt{25} = 5 \quad ; \text{ then}$$

$$\int_C \sqrt{x^2 + y^2} \, ds = \int_{-2\pi}^{2\pi} \sqrt{(4\cos t)^2 + (4\sin t)^2} \cdot \frac{ds}{dt} \, dt$$

$$= \int_{-2\pi}^{2\pi} \sqrt{16(\underbrace{\cos^2 t + \sin^2 t}_1)} (5) \, dt$$

$$= \int_{-2\pi}^{2\pi} (4)(5) \, dt = 20t \Big|_{-2\pi}^{2\pi}$$

$$= 20(2\pi) - 20(-2\pi) = 80\pi$$

13.) direction vector:

$$\begin{aligned}\vec{w} &= (0-1)\vec{i} + (-1-2)\vec{j} + (1-3)\vec{k} \\ &= (-1)\vec{i} + (-3)\vec{j} + (-2)\vec{k}, \text{ so}\end{aligned}$$

line L passing through points $(1, 2, 3)$ and $(0, -1, 1)$ is

$$L: \begin{cases} x = 1 - t \\ y = 2 - 3t \\ z = 3 - 2t \end{cases} \text{ for } 0 \leq t \leq 1;$$

then

$$\vec{r}(t) = (1-t)\vec{i} + (2-3t)\vec{j} + (3-2t)\vec{k} \xrightarrow{D}$$

$$\vec{v}(t) = (-1)\vec{i} + (-3)\vec{j} + (-2)\vec{k} \text{ and}$$

$$|\vec{v}(t)| = \sqrt{(-1)^2 + (-3)^2 + (-2)^2} = \sqrt{14}; \text{ then}$$

$$\int_C (x+y+z) ds = \int_0^1 (x+y+z) \frac{ds}{dt} dt$$

$$= \int_0^1 [(1-t) + (2-3t) + (3-2t)] \sqrt{14} dt$$

$$= \int_0^1 (6-6t) \sqrt{14} dt = \sqrt{14} (6t - 3t^2) \Big|_0^1$$

$$= 3\sqrt{14}$$

15.) $C_1: \vec{r}(t) = t\vec{i} + t^2\vec{j} \text{ for } 0 \leq t \leq 1 \xrightarrow{D}$

$$\vec{v}(t) = (1)\vec{i} + (2t)\vec{j} \text{ and}$$

$$|\vec{v}(t)| = \sqrt{(1)^2 + (2t)^2} = \sqrt{1+4t^2} ;$$

$$C_2: \vec{r}(t) = (1)\vec{i} + (1)\vec{j} + t\vec{k} \text{ for } 0 \leq t \leq 1 \xrightarrow{D}$$

$$\vec{v}(t) = (0)\vec{i} + (0)\vec{j} + (1)\vec{k} \text{ and}$$

$$|\vec{v}(t)| = \sqrt{0^2 + 0^2 + 1^2} = 1 ; \text{ now}$$

$$\int_C (x + \sqrt{y} - z^2) dS$$

$$= \int_{C_1} (x + \sqrt{y} - z^2) dS + \int_{C_2} (x + \sqrt{y} - z^2) dS$$

$$= \int_0^1 (t + \sqrt{t^2 - 0^2}) \frac{dS}{dt} dt + \int_0^1 (1 + \sqrt{1} - t^2) \frac{dS}{dt} dt$$

$$= \int_0^1 2t \cdot \sqrt{1+4t^2} dt + \int_0^1 (2 - t^2)(1) dt$$

$$= \frac{1}{4} \cdot \frac{2}{3} (1+4t^2)^{3/2} \Big|_0^1 + (2t - \frac{1}{3}t^3) \Big|_0^1$$

$$= \frac{1}{6} (5)^{3/2} - \frac{1}{6} (1)^{3/2} + (2 - \frac{1}{3})$$

$$= \frac{1}{6} (5)^{3/2} - \frac{1}{6} + \frac{12}{6} - \frac{2}{6} = \frac{9}{6} - \frac{1}{6} (5)^{3/2}$$

$$= \frac{3}{2} - \frac{1}{6} (5)^{3/2}$$

$$16.) C_1: \vec{r}(t) = t\vec{k} \text{ for } 0 \leq t \leq 1 \xrightarrow{D}$$

$$\vec{v}(t) = (1)\vec{k} \text{ and } |\vec{v}(t)| = 1$$

$$C_2: \vec{r}(t) = t\vec{j} + (1)\vec{k} \text{ for } 0 \leq t \leq 1 \xrightarrow{D}$$

$$\vec{v}(t) = (1)\vec{j} + (0)\vec{k} \text{ and}$$

$$|\vec{v}(t)| = \sqrt{1^2 + 0^2} = 1 ;$$

$$C_3: \vec{r}(t) = t\vec{i} + (1)\vec{j} + (1)\vec{k} \text{ for } 0 \leq t \leq 1 \xrightarrow{D}$$

$$\vec{v}(t) = (1)\vec{i} + (0)\vec{j} + (0)\vec{k} \text{ and}$$

$$|\vec{v}(t)| = \sqrt{1^2 + 0^2 + 0^2} = 1; \text{ then}$$

$$\begin{aligned} \int_C (x + \sqrt{y} - z^2) ds &= \int_{C_1} (x + \sqrt{y} - z^2) \frac{ds}{dx} dx \\ &+ \int_{C_2} (x + \sqrt{y} - z^2) \frac{ds}{dx} dx + \int_{C_3} (x + \sqrt{y} - z^2) \frac{ds}{dx} dx \\ &= \int_0^1 (0 + \sqrt{0} - t^2)(1) dx + \int_0^1 (0 + \sqrt{t} - 1^2)(1) dx \\ &+ \int_0^1 (t + \sqrt{1} - 1^2)(1) dx \\ &= \int_0^1 -t^2 dx + \int_0^1 (t^{\frac{1}{2}} - 1) dx + \int_0^1 t dx \\ &= -\frac{1}{3}t^3 \Big|_0^1 + \left(\frac{2}{3}t^{\frac{3}{2}} - t\right) \Big|_0^1 + \frac{1}{2}t^2 \Big|_0^1 \\ &= -\frac{1}{3} + \frac{2}{3} - 1 + \frac{1}{2} = -\frac{2}{3} + \frac{1}{2} = -\frac{4}{6} + \frac{3}{6} = -\frac{1}{6} \end{aligned}$$

17.) $\vec{r}(t) = t\vec{i} + t\vec{j} + t\vec{k}$ for $0 < a \leq t \leq b \xrightarrow{D}$

$$\vec{v}(t) = (1)\vec{i} + (1)\vec{j} + (1)\vec{k} \text{ and}$$

$$|\vec{v}(t)| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}; \text{ then}$$

$$\int_C \frac{x+y+z}{x^2+y^2+z^2} ds = \int_a^b \frac{t+t+t}{t^2+t^2+t^2} \frac{ds}{dt} dt$$

$$= \int_a^b \frac{3t}{3t^2} \sqrt{3} dt = \sqrt{3} \int_a^b \frac{1}{t} dt$$

$$= \sqrt{3} \ln|t| \Big|_a^b = \sqrt{3} (\ln b - \ln a) = \sqrt{3} \ln\left(\frac{b}{a}\right).$$

$$18.) \vec{r}(t) = (a \cos t) \vec{j} + (a \sin t) \vec{k}$$

$$\text{for } 0 \leq t \leq 2\pi \xrightarrow{D}$$

$$\vec{v}(t) = (-a \sin t) \vec{j} + (a \cos t) \vec{k} \text{ and}$$

$$|\vec{v}(t)| = \sqrt{(-a \sin t)^2 + (a \cos t)^2}$$

$$= \sqrt{a^2(\underbrace{\sin^2 t + \cos^2 t}_1)} = a \quad (a > 0);$$

$$\int_C -\sqrt{x^2 + z^2} \, ds = - \int_0^{2\pi} \sqrt{0^2 + (a \sin t)^2} \frac{ds}{dt} dt$$

$$= - \int_0^{2\pi} \sqrt{a^2 \sin^2 t} (a) dt$$

$$= -a^2 \int_0^{2\pi} |\sin t| dt$$

$$= -a^2 \cdot 2 \int_0^{\pi} \sin t \, dt$$

$$= -2a^2 (-\cos t) \Big|_0^{\pi} = 2a^2 (\cos \pi - \cos 0)$$

$$= 2a^2 (-1 - 1) = -4a^2.$$

$$27.) C: y = \frac{1}{2}x^2 \rightarrow \begin{cases} x = t \\ y = \frac{1}{2}t^2 \end{cases} \text{ for } 0 \leq x \leq 2 \rightarrow 0 \leq t \leq 2$$

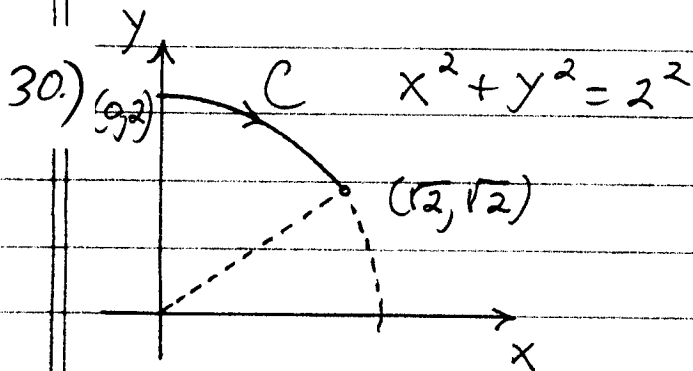
$$\rightarrow \vec{r}(t) = t \vec{i} + \frac{1}{2}t^2 \vec{j} \xrightarrow{D}$$

$$\vec{v}(t) = (1) \vec{i} + t \vec{j} \text{ and}$$

$$|\vec{v}(t)| = \sqrt{1 + t^2}; \text{ then}$$

$$\int_C \frac{x^3}{y} \, ds = \int_0^2 \frac{t^3}{\frac{1}{2}t^2} \cdot \frac{ds}{dt} dt$$

$$\begin{aligned}
 &= 2 \int_0^2 t \sqrt{1+t^2} dx = \frac{2}{3} (1+t^2)^{3/2} \Big|_0^2 \\
 &= \frac{2}{3} (5)^{3/2} - \frac{2}{3} (1)^{3/2} = \frac{2}{3} (5)^{3/2} - \frac{2}{3}
 \end{aligned}$$



$$\begin{aligned}
 C: &\begin{cases} x = 2 \sin t \\ y = 2 \cos t \end{cases} \\
 &\text{for } 0 \leq t \leq \frac{\pi}{4} ;
 \end{aligned}$$

$$\vec{r}(t) = (2 \sin t) \vec{i} + (2 \cos t) \vec{j} \xrightarrow{D}$$

$$\vec{v}(t) = (2 \cos t) \vec{i} + (-2 \sin t) \vec{j} \quad \text{and}$$

$$|\vec{v}(t)| = \sqrt{(2 \cos t)^2 + (-2 \sin t)^2}$$

$$= \sqrt{4 (\underbrace{\cos^2 t + \sin^2 t}_1)} = 2 ; \text{ then}$$

$$\int_C (x^2 y) ds = \int_0^{\pi/4} ((2 \sin t)^2 - 2 \cos t) \frac{ds}{dt} dt$$

$$= \int_0^{\pi/4} (4 \sin^2 t - 2 \cos t) (2) dt$$

$$= 4 \int_0^{\pi/4} (2 \sin^2 t - \cos t) dt$$

$$= 4 \int_0^{\pi/4} \left[2 \cdot \frac{1}{2} (1 - \cos 2t) - \cos t \right] dt$$

$$= 4 \left(t - \frac{1}{2} \sin 2t - \sin t \right) \Big|_0^{\pi/4}$$

$$= 4 \left(\frac{\pi}{4} - \frac{1}{2} \sin \frac{\pi}{2} - \sin \frac{\pi}{4} \right)$$

$$- 4 \left(0 - \frac{1}{2} \sin 0 - \sin 0 \right)$$

$$= 4 \left(\frac{\pi}{4} - \frac{1}{2}(1) - \frac{\sqrt{2}}{2} \right)$$

$$= \pi - 2 - 2\sqrt{2}$$

33) $C: \vec{r}(t) = (t^2-1)\vec{j} + (2t)\vec{k}$ for $0 \leq t \leq 1$ $\xrightarrow{D}$

$\vec{v}(t) = (2t)\vec{j} + (2)\vec{k}$ and

$$|\vec{v}(t)| = \sqrt{(2t)^2 + 2^2} = \sqrt{4t^2 + 4} = 2\sqrt{t^2 + 1};$$

density $\delta(x, y, z) = \frac{3}{2}t$;

$$\text{Mass} = \int_C \delta(P) ds = \int_0^1 \frac{3}{2}t \cdot \frac{ds}{dt} dt$$

$$= \int_0^1 \frac{3}{2}t \cdot 2\sqrt{t^2+1} dt = 3 \int_0^1 t\sqrt{t^2+1} dt$$

$$= 3 \cdot \frac{1}{2} \cdot \frac{2}{3} (t^2+1)^{3/2} \Big|_0^1 = 2^{3/2} - 1^{3/2}$$

$$= 2^{3/2} - 1$$

34) $C: \vec{r}(t) = (t^2-1)\vec{j} + (2t)\vec{k}$ for $-1 \leq t \leq 1$

$\xrightarrow{D} \vec{v}(t) = (2t)\vec{j} + (2)\vec{k}$ and

$$|\vec{v}(t)| = \sqrt{(2t)^2 + (2)^2} = \sqrt{4t^2 + 4} = 2\sqrt{t^2 + 1};$$

density $\delta(x, y, z) = 15\sqrt{y+2}$, then

$$\bar{y} = \frac{\int_C y \cdot \delta(P) ds}{\int_C \delta(P) ds};$$

$$\int_C \delta(P) ds = \int_{-1}^1 15 \sqrt{(t^2-1)+2} \cdot \frac{ds}{dt} dt$$

$$= 15 \int_{-1}^1 \sqrt{t^2+1} \cdot 2 \sqrt{t^2+1} dt$$

$$= 30 \int_{-1}^1 (t^2+1) dt = 30 \left(\frac{1}{3} t^3 + t \right) \Big|_{-1}^1$$

$$= 30 \left(\frac{1}{3} + 1 \right) - 30 \left(-\frac{1}{3} - 1 \right) = \frac{10}{3} (4) - \frac{10}{3} (-4)$$

$$= 80$$

$$\int_C y \cdot \delta(P) dt = \int_{-1}^1 (t^2-1) \sqrt{t^2+1} \cdot 2 \sqrt{t^2+1} dt$$

$$= 2 \int_{-1}^1 (t^2-1)(t^2+1) dt = 2 \int_{-1}^1 (t^4-1) dt$$

$$= 2 \left(\frac{1}{5} t^5 - t \right) \Big|_{-1}^1 = 2 \left(\frac{1}{5} - 1 \right) - 2 \left(-\frac{1}{5} + 1 \right)$$

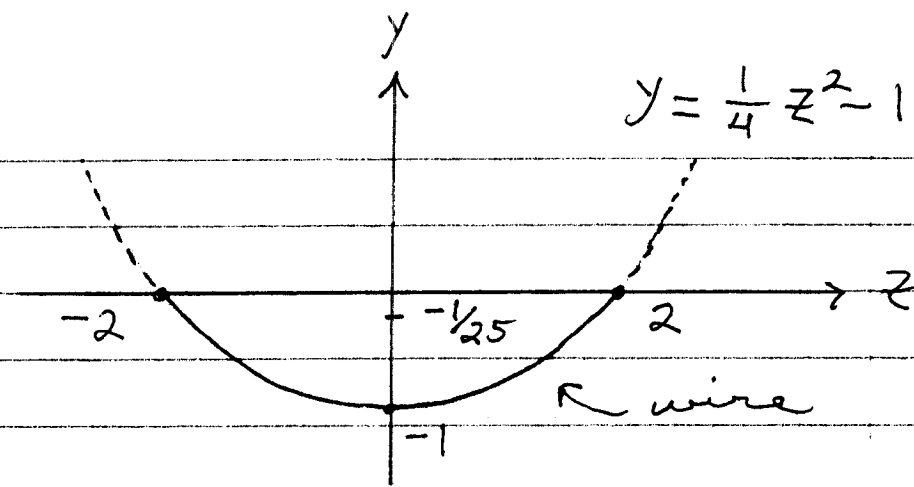
$$= 2 \left(-\frac{4}{5} \right) - 2 \left(\frac{4}{5} \right) = -\frac{16}{5} \text{ so}$$

$$\bar{y} = \frac{-\frac{16}{5}}{\frac{80}{1}} = -\frac{16}{5} \cdot \frac{1}{80} = \left(\frac{-1}{25} \right)$$

$$\begin{cases} y = t^2 - 1 \\ z = 2t \end{cases} \text{ for } -1 \leq t \leq 1 \rightarrow$$

$$t = \frac{1}{2} z \rightarrow (\text{sub}) \rightarrow y = \left(\frac{1}{2} z \right)^2 - 1 \rightarrow$$

$$y = \frac{1}{4} z^2 - 1$$



35.) a.) $C: \vec{r}(t) = \sqrt{2}t \vec{i} + \sqrt{2}t \vec{j} + (4-t^2) \vec{k}$
for $0 \leq t \leq 1$ $\xrightarrow{D}$

$\vec{v}(t) = \sqrt{2} \vec{i} + \sqrt{2} \vec{j} + (-2t) \vec{k}$ then

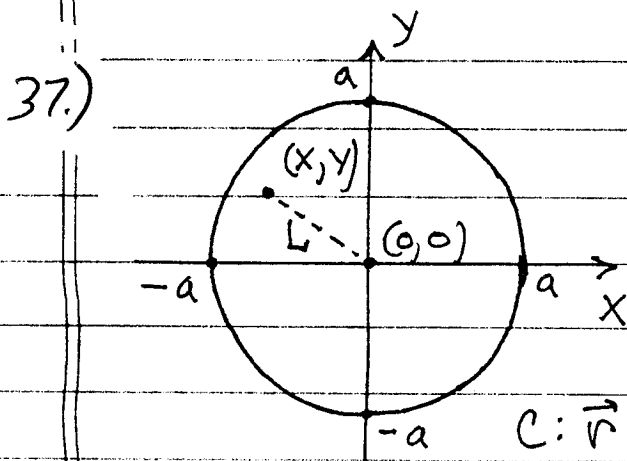
$$|\vec{v}(t)| = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2 + (-2t)^2} = \sqrt{4 + 4t^2}$$

$$= 2\sqrt{1+t^2} \quad ; \text{ density } \delta(x,y,z) = 3t \rightarrow$$

$$\text{Mass} = \int_C \delta(P) ds = \int_0^1 3t \cdot \frac{ds}{dt} dt$$

$$= \int_0^1 3t \cdot 2\sqrt{1+t^2} dt = 2(1+t^2)^{3/2} \Big|_0^1$$

$$= 2(2)^{3/2} - 2(1)^{3/2} = 4\sqrt{2} - 2$$



density is constant
value δ ;

distance

$$L = \sqrt{x^2 + y^2} \quad ;$$

$$C: \vec{r}(t) = a \cos t \vec{i} + a \sin t \vec{j}$$

for $0 \leq t \leq 2\pi$ $\xrightarrow{D}$ $\vec{v}(t) = -a \sin t \vec{i} + a \cos t \vec{j}$

$$\rightarrow |\vec{v}(t)| = \sqrt{(-a \sin t)^2 + (a \cos t)^2} = \dots = a \quad ;$$

$$M. \text{ of } I. = \int_C (\text{distance})^2 \cdot \delta(P) ds$$

$$= \int_0^{2\pi} (x^2 + y^2) \cdot \delta \frac{ds}{dt} dt$$

$$= \int_0^{2\pi} (a \cos t)^2 + (a \sin t)^2 \cdot \delta \cdot (a) dt$$

$$= \int_0^{2\pi} (a^2 (\underbrace{\cos^2 t + \sin^2 t}_1)) \delta a dt$$

$$= a^3 \delta t \Big|_0^{2\pi} = 2\pi a^3 \delta$$

39.) $C : \vec{r}(t) = (\cos t) \vec{i} + (\sin t) \vec{j} + t \vec{k}$

for $0 \leq t \leq 2\pi$ $\xrightarrow{D}$

$$\vec{v}(t) = (-\sin t) \vec{i} + (\cos t) \vec{j} + (1) \vec{k} \text{ and}$$

$$|\vec{v}(t)| = \sqrt{(-\sin t)^2 + (\cos t)^2 + (1)^2}$$

$$= \sqrt{1+1} = \sqrt{2} ; \text{ assume}$$

density is $\delta(x, y, z) = z$; then

$$a.) I_z = \int_C (\text{distance})^2 \delta(P) ds$$

from (x, y, z) to $(0, 0, z)$

$$= \int_0^{2\pi} (x^2 + y^2) \cdot z \frac{ds}{dt} dt$$

$$= \int_0^{2\pi} (\underbrace{\cos^2 t + \sin^2 t}_1) \cdot t \cdot \sqrt{2} dt$$

$$= \sqrt{2} \cdot \frac{1}{2} t^2 \Big|_0^{2\pi} = \frac{\sqrt{2}}{2} \cdot 4\pi^2 = 2\sqrt{2} \pi^2$$

b.) M. of I. = $\int_C (\text{distance})^2 \cdot \delta(P) ds$
 from (x, y, z) to $(0, 0, 0)$

$$= \int_0^{2\pi} (x^2 + y^2 + z^2) \cdot z \cdot \frac{ds}{dt} dt$$

$$= \int_0^{2\pi} (\underbrace{\cos^2 t + \sin^2 t}_1 + t^2) t \cdot \sqrt{2} ds$$

$$= \sqrt{2} \int_0^{2\pi} (t + t^3) dt = \sqrt{2} \left(\frac{1}{2} t^2 + \frac{1}{4} t^4 \right) \Big|_0^{2\pi}$$

$$= \sqrt{2} \left(\frac{1}{2} (4\pi^2) + \frac{1}{4} (16\pi^4) \right)$$

$$= \sqrt{2} (2\pi^2 + 4\pi^4)$$

c.) $\bar{z} = \frac{\int_C z \delta(P) ds}{\int_C \delta(P) ds}$;

$$\int_C \delta(P) ds = \int_0^{2\pi} z \cdot \frac{ds}{dt} dt = \int_0^{2\pi} t \cdot \sqrt{2} dt$$

$$= \sqrt{2} \cdot \frac{1}{2} t^2 \Big|_0^{2\pi} = \sqrt{2} \cdot \frac{1}{2} (4\pi^2) = 2\sqrt{2} \pi^2 ;$$

$$\int_C z \delta(P) ds = \int_0^{2\pi} t \cdot z \cdot \frac{ds}{dt} dt$$

$$= \int_0^{2\pi} t \cdot t \cdot \sqrt{2} \, dt = \sqrt{2} \int_0^{2\pi} t^2 \, dt$$

$$= \sqrt{2} \cdot \frac{1}{3} t^3 \Big|_0^{2\pi} = \frac{\sqrt{2}}{3} \cdot 8\pi^3 = \frac{8}{3} \sqrt{2} \pi^3 ;$$

then

$$\bar{z} = \frac{\frac{8}{3} \sqrt{2} \pi^3}{2\sqrt{2} \pi^2} = \frac{4}{3} \pi$$

$$d.) \bar{z} = \frac{\int_C 1 \cdot \delta(P) \, ds}{\int_C 1 \, ds} ;$$

$$\int_C 1 \, ds = \int_0^{2\pi} \frac{ds}{dt} \cdot dt = \int_0^{2\pi} \sqrt{2} \, dt$$

$$= \sqrt{2} t \Big|_0^{2\pi} = 2\sqrt{2} \pi ;$$

$$\int_C \delta(P) \, ds = \dots = 2\sqrt{2} \pi^2 ; \text{ then}$$

$$\bar{z} = \frac{2\sqrt{2} \pi^2}{2\sqrt{2} \pi} = \pi$$