

Section 16.2

29.) a.) $\vec{F}_1(x, y) = (x)\vec{i} + (y)\vec{j}$ and

$$C: \begin{cases} x = \cos t \\ y = \sin t \end{cases} \text{ for } 0 \leq t \leq 2\pi$$

$$\text{and } \frac{dx}{dt} = -\sin t, \frac{dy}{dt} = \cos t;$$

$$\text{Circ}_1 = \int_C \vec{F}_1 \cdot \vec{T} \, ds = \int_C M \, dx + N \, dy$$

$$= \int_0^{2\pi} \left[x \cdot \frac{dx}{dt} + y \cdot \frac{dy}{dt} \right] dt$$

$$= \int_0^{2\pi} [\cos t \cdot (-\sin t) + \sin t \cdot \cos t] dt$$

$$= \int_0^{2\pi} 0 \, dt = 0;$$

$$\text{Flux}_1 = \int_C \vec{F}_1 \cdot \vec{n} \, ds = \int_C M \, dy - N \, dx$$

$$= \int_0^{2\pi} \left[x \cdot \frac{dy}{dt} - y \cdot \frac{dx}{dt} \right] dt$$

$$= \int_0^{2\pi} [\cos t \cdot \cos t - \sin t \cdot (-\sin t)] dt$$

$$= \int_0^{2\pi} (\underbrace{\cos^2 t + \sin^2 t}_1) dt = t \Big|_0^{2\pi} = 2\pi$$

$\vec{F}_2(x, y) = (-y)\vec{i} + (x)\vec{j}$ then

$$\text{Circ}_2 = \int_C \vec{F}_2 \cdot \vec{T} \, ds = \int_C M \, dx + N \, dy$$

$$= \int_0^{2\pi} \left[-y \cdot \frac{dx}{dt} + x \cdot \frac{dy}{dt} \right] dt$$

$$\begin{aligned}
 &= \int_0^{2\pi} [-\sin t \cdot -\sin t + \cos t \cdot \cos t] dt \\
 &= \int_0^{2\pi} [\underbrace{\sin^2 t + \cos^2 t}_1] dt = t \Big|_0^{2\pi} = 2\pi ;
 \end{aligned}$$

$$\begin{aligned}
 \text{Flux}_2 &= \int_{C_2} \vec{F} \cdot \vec{n} ds = \int_{C_2} M dy - N dx \\
 &= \int_0^{2\pi} \left[-y \cdot \frac{dy}{dt} - x \cdot \frac{dx}{dt} \right] dt \\
 &= \int_0^{2\pi} [-\sin t \cdot -\cos t - \cos t \cdot -\sin t] dt \\
 &= \int_0^{2\pi} 2 \sin t \cos t dt = \sin^2 t \Big|_0^{2\pi} \\
 &= \cancel{\sin^2 2\pi} - \cancel{\sin^2 0} = 0 .
 \end{aligned}$$

30.) $C: \begin{cases} x = a \cos t \\ y = a \sin t \end{cases} \text{ for } 0 \leq t \leq 2\pi ,$

$$\frac{dx}{dt} = -a \sin t, \quad \frac{dy}{dt} = a \cos t ; \text{ then}$$

$$\vec{F}_1(x, y) = (2x)\vec{i} + (-3y)\vec{j} \quad \text{so}$$

$$\text{Flux}_1 = \int_C \vec{F}_1 \cdot \vec{n} ds = \int_C M dy - N dx$$

$$= \int_0^{2\pi} \left[2x \cdot \frac{dy}{dt} - (-3y) \frac{dx}{dt} \right] dt$$

$$= \int_0^{2\pi} [2a \cos t \cdot a \cos t + 3a \sin t \cdot -a \sin t] dt$$

$$= \int_0^{2\pi} [2a^2 \cos^2 t - 3a^2 \sin^2 t] dt$$

$$= \int_0^{2\pi} \left[2a^2 \cdot \frac{1}{2}(1 + \cos 2t) - 3a^2 \cdot \frac{1}{2}(1 - \cos 2t) \right] dt$$

$$\begin{aligned}
&= \left[a^2 \left(t + \frac{1}{2} \sin 2t \right) - \frac{3}{2} a^2 \left(t - \frac{1}{2} \sin 2t \right) \right] \Big|_0^{2\pi} \\
&= \left[a^2 \left(2\pi + \frac{1}{2} \sin 4\pi \right) - \frac{3}{2} a^2 \left(2\pi - \frac{1}{2} \sin 4\pi \right) \right] \\
&\quad - \left[a^2 \left(0 + \frac{1}{2} \sin 0 \right) - \frac{3}{2} a^2 \left(0 - \frac{1}{2} \sin 0 \right) \right] \\
&= 2a^2 \pi - 3a^2 \pi = -a^2 \pi ;
\end{aligned}$$

$$\vec{F}_2(x, y) = (2x)\vec{i} + (x-y)\vec{j} \quad \text{then}$$

$$\begin{aligned}
\text{Flux}_2 &= \int_C \vec{F}_2 \cdot \vec{n} \, ds = \int_C M \, dy - N \, dx \\
&= \int_0^{2\pi} \left[2x \cdot \frac{dy}{dt} - (x-y) \cdot \frac{dx}{dt} \right] dt \\
&= \int_0^{2\pi} \left[2 \cdot a \cos t \cdot a \cos t - (a \cos t - a \sin t) \cdot (-a \sin t) \right] dt \\
&= \int_0^{2\pi} \left[2 \cdot a^2 \cos^2 t + a^2 \sin t \cos t - a^2 \sin^2 t \right] dt \\
&= \int_0^{2\pi} \left[2a^2 \cdot \frac{1}{2} (1 + \cos 2t) + a^2 \sin t \cos t - a^2 \cdot \frac{1}{2} (1 - \cos 2t) \right] dt \\
&= \left[a^2 \left(t + \frac{1}{2} \sin 2t \right) + a^2 \frac{1}{2} \sin^2 t - \frac{a^2}{2} \left(t - \frac{1}{2} \sin 2t \right) \right] \Big|_0^{2\pi} \\
&= \left[a^2 \left(2\pi + \frac{1}{2} \sin 4\pi \right) + \frac{a^2}{2} \sin^2 2\pi - \frac{a^2}{2} \left(2\pi - \frac{1}{2} \sin 4\pi \right) \right] \\
&\quad - \left[a^2 \left(0 + \frac{1}{2} \sin 0 \right) + \frac{a^2}{2} \sin^2 0 - \frac{a^2}{2} \left(0 - \frac{1}{2} \sin 0 \right) \right] \\
&= 2a^2 \pi - a^2 \pi = a^2 \pi .
\end{aligned}$$

32.) $C_1: \begin{cases} x = a \cos t \\ y = a \sin t \end{cases} \text{ for } 0 \leq t \leq \pi$

$C_2: \begin{cases} x = t - a \\ y = 0 \end{cases} \text{ for } 0 \leq t \leq 2a$;

$\vec{F}(x,y) = (x^2)\vec{i} + (y^2)\vec{j}$ then

Circ = $\int_C \vec{F} \cdot \vec{T} ds = \int_C M dx + N dy$

$= \int_{C_1} M dx + N dy + \int_{C_2} M dx + N dy$

$= \int_0^\pi \left[x^2 \frac{dx}{dt} + y^2 \frac{dy}{dt} \right] dt + \int_0^{2a} \left[x^2 \frac{dx}{dt} + y^2 \frac{dy}{dt} \right] dt$

$= \int_0^\pi [a^2 \cos^2 t \cdot -a \sin t + a^2 \sin^2 t \cdot a \cos t] dt$

$+ \int_0^{2a} [(t-a)^2 (1) + (0)^2 (0)] dt$

$= \int_0^\pi [-a^3 \cos^2 t \sin t + a^3 \sin^2 t \cos t] dt$

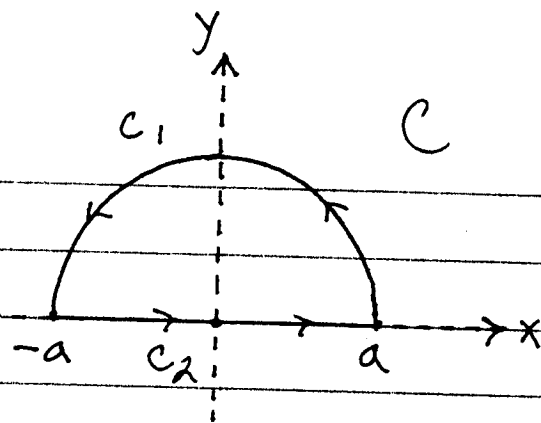
$+ \frac{1}{3} (t-a)^3 \Big|_0^{2a}$

$= \left(-a^3 \cdot -\frac{1}{3} \cos^3 t + a^3 \cdot \frac{1}{3} \sin^3 t \right) \Big|_0^\pi + \frac{1}{3} a^3 - \frac{1}{3} a^3$

$= \left(\frac{1}{3} a^3 \cos^3 \pi + \frac{1}{3} a^3 \sin^3 \pi \right) - \left(\frac{1}{3} a^3 \cos^3 0 + \frac{1}{3} a^3 \sin^3 0 \right)$

$= \frac{1}{3} a^3 (-1)^3 - \frac{1}{3} a^3 (1)^3 + \frac{2}{3} a^3 = -\frac{2}{3} a^3 + \frac{2}{3} a^3 = 0 ;$

Flux = $\int_C \vec{F} \cdot \vec{n} ds = \int_C M dy - N dx$



$$\begin{aligned}
&= \int_{C_1} M dy - N dx + \int_{C_2} M dy - N dx \\
&= \int_0^\pi \left[x^2 \frac{dy}{dt} - y^2 \frac{dx}{dt} \right] dt + \int_0^{2a} \left[x^2 \frac{dy}{dt} - 0 \cdot \frac{dx}{dt} \right] dt \\
&= \int_0^\pi \left[a^2 \cos^2 t \cdot a \cos t - a^2 \sin^2 t \cdot (-a \sin t) \right] dt \\
&= \int_0^\pi (a^3 \cos^3 t + a^3 \sin^3 t) dt \\
&= a^3 \int_0^\pi [\cos^2 t \cdot \cos t + \sin^2 t \cdot \sin t] dt \\
&= a^3 \int_0^\pi [(1 - \sin^2 t) \cos t + (1 - \cos^2 t) \sin t] dt \\
&= a^3 \int_0^\pi [\cos t - \sin^2 t \cos t + \sin t - \cos^2 t \sin t] dt \\
&= a^3 \left[\sin t - \frac{1}{3} \sin^3 t - \cos t + \frac{1}{3} \cos^3 t \right] \Big|_0^\pi \\
&= a^3 \left(\overset{0}{\sin \pi} - \frac{1}{3} \overset{0}{\sin^3 \pi} - \cos \pi + \frac{1}{3} \cos^3 \pi \right) \\
&\quad - a^3 \left(\overset{0}{\sin 0} - \frac{1}{3} \overset{0}{\sin^3 0} - \cos 0 + \frac{1}{3} \cos^3 0 \right) \\
&= a^3 \left(-(-1) + \frac{1}{3} (-1)^3 \right) - a^3 \left(-1 + \frac{1}{3} (1)^3 \right) \\
&= a^3 \left(\frac{2}{3} \right) - a^3 \left(-\frac{2}{3} \right) = \frac{4}{3} a^3.
\end{aligned}$$

33) $\vec{F}(x, y) = (-y) \vec{i} + (x) \vec{j}$ then

$$\begin{aligned}
\text{Circ} &= \int_C \vec{F} \cdot \vec{T} ds = \int_C M dx + N dy \\
&= \int_{C_1} M dx + N dy + \int_{C_2} M dx + N dy \\
&= \int_0^\pi \left[(-y) \frac{dx}{dt} + x \frac{dy}{dt} \right] dt + \int_0^{2a} \left[(-y) \frac{dx}{dt} + x \frac{dy}{dt} \right] dt \\
&= \int_0^\pi [-a \sin t \cdot (-a \sin t) + a \cos t \cdot a \cos t] dt \\
&\quad + \int_0^{2a} [(0)(1) + (t-a)(0)] dt
\end{aligned}$$

$$\begin{aligned}
 &= \int_0^{\pi} [a^2 \sin^2 t + a^2 \cos^2 t] dt \\
 &= a^2 \int_0^{\pi} (\sin^2 t + \cos^2 t) dt \\
 &= a^2 \cdot t \Big|_0^{\pi} = a^2 \pi
 \end{aligned}$$

$$\text{Flux} = \int_C \vec{F} \cdot \vec{n} \, ds = \int_C M \, dy - N \, dx$$

$$= \int_{C_1} M \, dy - N \, dx + \int_{C_2} M \, dy - N \, dx$$

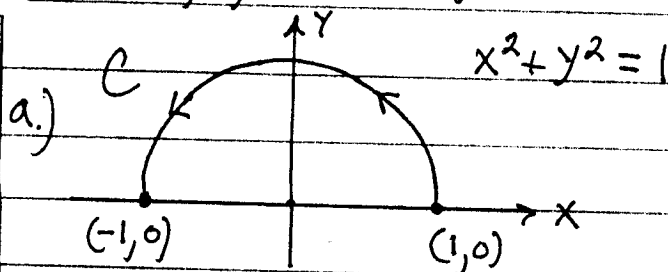
$$= \int_0^{\pi} \left[(-y) \cdot \frac{dy}{dt} - x \cdot \frac{dx}{dt} \right] dt + \int_0^{2a} \left[(y) \cdot \frac{dy}{dt} - x \cdot \frac{dx}{dt} \right] dt$$

$$\begin{aligned}
 &= \int_0^{\pi} [-a \sin t \cdot a \cos t - a \cos t \cdot -a \sin t] dt \\
 &\quad + \int_0^{2a} [(0)(0) - (t-a)(1)] dt
 \end{aligned}$$

$$\begin{aligned}
 &= \int_0^{\pi} (-a^2 \sin t \cos t + a^2 \sin t \cos t) dt \\
 &\quad + \int_0^{2a} (a - t) dt
 \end{aligned}$$

$$= \left(at - \frac{1}{2} t^2 \right) \Big|_0^{2a} = 2a^2 - 2a^2 = 0$$

35.) $\vec{F}(x,y) = (x+y) \vec{i} - (x^2+y^2) \vec{j}$

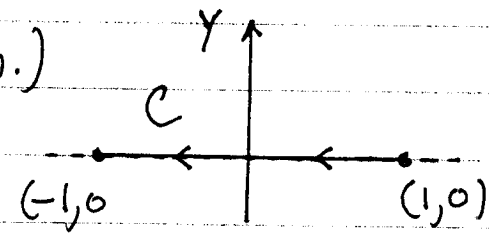


$$C: \begin{cases} x = \cos t \\ y = \sin t \end{cases} \text{ for } 0 \leq t \leq \pi$$

$$\frac{dx}{dt} = -\sin t, \quad \frac{dy}{dt} = \cos t$$

$$\begin{aligned}
\text{Flow} &= \int_C \vec{F} \cdot \vec{T} \, ds = \int_C M \, dx + N \, dy \\
&= \int_0^\pi \left[(x+y) \frac{dx}{dt} + (-x^2-y^2) \frac{dy}{dt} \right] dt \\
&= \int_0^\pi \left[(\cos t + \sin t) \cdot (-\sin t) + (-\cos^2 t - \sin^2 t) \cdot \cos t \right] dt \\
&= \int_0^\pi \left[-\sin t \cos t - \sin^2 t - \cos^2 t \right] dt \\
&= \int_0^\pi \left[-\sin t \cdot \cos t - \frac{1}{2}(1 - \cos 2t) - \cos t \right] dt \\
&= \left(-\frac{1}{2} \sin^2 t - \frac{1}{2} \left(t - \frac{1}{2} \sin 2t \right) - \sin t \right) \Big|_0^\pi \\
&= \left(-\frac{1}{2} \sin^2 \pi - \frac{1}{2} \pi + \frac{1}{4} \sin^2 2\pi - \sin \pi \right) \\
&\quad - \left(-\frac{1}{2} \sin^2 0 - \frac{1}{2} (0) + \frac{1}{4} \sin^2 0 - \sin 0 \right) = -\frac{1}{2} \pi.
\end{aligned}$$

b.)

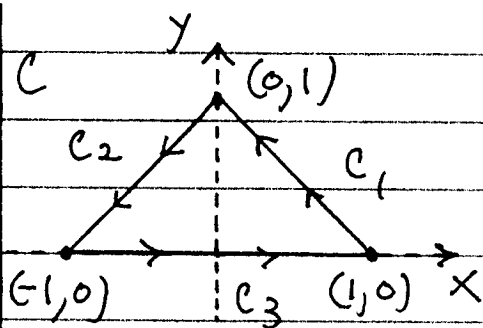


$$C: \begin{cases} x = 1-t & \text{for} \\ y = 0 & 0 \leq t \leq 2 \end{cases}$$

$$\frac{dx}{dt} = -1, \quad \frac{dy}{dt} = 0$$

$$\begin{aligned}
\text{Flow} &= \int_C \vec{F} \cdot \vec{T} \, ds = \int_C M \, dx + N \, dy \\
&= \int_0^2 \left[(x+y) \frac{dx}{dt} + (-x^2-y^2) \frac{dy}{dt} \right] dt \\
&= \int_0^2 \left[(1-t) \cdot (-1) + (- (1-t)^2 - 0^2) (0) \right] dt \\
&= \int_0^2 (t-1) \, dt = \left(\frac{1}{2} t^2 - t \right) \Big|_0^2 \\
&= 2 - 2 = 0
\end{aligned}$$

36.) $\vec{F}(x,y) = (x+y)\vec{i} - (x^2+y^2)\vec{j}$



$$C_1: \begin{cases} x=t & \text{for } t=-1 \text{ to } t=0 \\ y=1-t & \end{cases}, \quad \frac{dx}{dt}=1, \quad \frac{dy}{dt}=-1$$

$$C_2: \begin{cases} x=t & \text{for } t=0 \text{ to } t=-1 \\ y=t+1 & \end{cases}, \quad \frac{dx}{dt}=1, \quad \frac{dy}{dt}=1$$

$$C_3: \begin{cases} x=t & \text{for } -1 \leq x \leq 1 \\ y=0 & \end{cases}, \quad \frac{dx}{dt}=1, \quad \frac{dy}{dt}=0$$

$$\text{Flux} = \int_C \vec{F} \cdot \vec{n} \, ds$$

$$= \int_{C_1} \vec{F} \cdot \vec{n} \, ds + \int_{C_2} \vec{F} \cdot \vec{n} \, ds + \int_{C_3} \vec{F} \cdot \vec{n} \, ds$$

$$\int_{C_1} \vec{F} \cdot \vec{n} \, ds = \int_{C_1} M \, dy - N \, dx = \int_{-1}^0 [(x+y) \frac{dy}{dt} + (x^2+y^2) \frac{dx}{dt}] dt$$

$$= \int_{-1}^0 [(1)(-1) + (t^2 + (1-t)^2)(1)] dt$$

$$= \int_{-1}^0 [-1 + t^2 + 1 - 2t + t^2] dt = \int_{-1}^0 [2t^2 - 2t] dt$$

$$= \left(\frac{2}{3} t^3 - t^2 \right) \Big|_{-1}^0 = (0 - 0) - \left(\frac{2}{3} - 1 \right) = \frac{1}{3}$$

$$\int_{C_2} \vec{F} \cdot \vec{n} \, ds = \int_{C_2} M \, dy - N \, dx = \int_0^{-1} [(x+y) \frac{dy}{dt} + (x^2+y^2) \frac{dx}{dt}] dt$$

$$= \int_0^{-1} [(2t+1)(1) + (t^2 + (t+1)^2)(1)] dt$$

$$= \int_0^{-1} [2t+1+t^2+t^2+2t+1] dt$$

$$= \int_0^{-1} (2t^2 + 4t + 2) dt = \left(\frac{2}{3} t^3 + 2t^2 + 2t \right) \Big|_0^{-1}$$

$$= \left(-\frac{2}{3} + 2 - 2 \right) - (0 + 0 + 0) = -\frac{2}{3} ;$$

$$\int_{C_3} \vec{F} \cdot \vec{n} ds = \int_{C_3} M dy - N dx = \int_{-1}^1 \left[(x+y) \frac{dy}{dx} + (x^2+y^2) \frac{dx}{dx} \right] dx$$

$$= \int_{-1}^1 [(t) \cdot (0) + (t^2)(1)] dt = \frac{1}{3} t^3 \Big|_{-1}^1 = \frac{1}{3} - \frac{-1}{3} = \frac{2}{3} ;$$

$$\text{Flux} = \frac{1}{3} + \frac{-2}{3} + \frac{2}{3} = \frac{1}{3}$$

$$47.) \vec{F}(x, y, z) = (-4xy)\vec{i} + (8y)\vec{j} + (2)\vec{k} ,$$

$$C: \begin{cases} x=t \\ y=t^2 \\ z=1 \end{cases} \text{ for } 0 \leq t \leq 2; \quad \frac{dx}{dt}=1, \frac{dy}{dt}=2t, \frac{dz}{dt}=0;$$

$$\text{Flow} = \int_C \vec{F} \cdot \vec{T} ds = \int_C M dx + N dy + P dz$$

$$= \int_0^2 \left[-4xy \cdot \frac{dx}{dt} + 8y \cdot \frac{dy}{dt} + 2 \cdot \frac{dz}{dt} \right] dt$$

$$= \int_0^2 [-4(t)(t^2)(1) + 8(t^2)(2t)] dt$$

$$= \int_0^2 12t^3 dt = 3t^4 \Big|_0^2 = 48$$

$$50.) \vec{F}(x, y, z) = (-y)\vec{i} + (x)\vec{j} + (2)\vec{k}$$

$$C: \begin{cases} x = -2 \cos t \\ y = 2 \sin t \\ z = 2t \end{cases} \text{ for } 0 \leq t \leq 2\pi$$

$$\frac{dx}{dt} = 2 \sin t, \quad \frac{dy}{dt} = 2 \cos t, \quad \frac{dz}{dt} = 2;$$

$$\text{Flow} = \int_C \vec{F} \cdot \vec{T} \, ds = \int_C M dx + N dy + P dz$$

$$= \int_0^{2\pi} \left[(-y) \frac{dx}{dt} + (x) \frac{dy}{dt} + (2) \frac{dz}{dt} \right] dt$$

$$= \int_0^{2\pi} \left[(-2 \sin t)(2 \sin t) + (-2 \cos t)(2 \cos t) + (2)(2) \right] dt$$

$$= \int_0^{2\pi} \left[-4 \sin^2 t - 4 \cos^2 t + 4 \right] dt$$

$$= \int_0^{2\pi} \left[-4 (\underbrace{\sin^2 t + \cos^2 t}_1) + 4 \right] dt$$

$$= \int_0^{2\pi} 0 \, dt = 0.$$

51.) $\vec{F}(x, y, z) = (2x)\vec{i} + (2z)\vec{j} + (2y)\vec{k},$

$$C_1: \begin{cases} x = \cos t \\ y = \sin t \\ z = t \end{cases} \text{ for } 0 \leq t \leq \frac{\pi}{2},$$

$$\frac{dx}{dt} = -\sin t, \quad \frac{dy}{dt} = \cos t, \quad \frac{dz}{dt} = 1;$$

$$C_2: \begin{cases} x = 0 \\ y = 1 \\ z = \frac{\pi}{2}(1-t) \end{cases} \text{ for } 0 \leq t \leq 1,$$

$$\frac{dx}{dt} = 0, \quad \frac{dy}{dt} = 0, \quad \frac{dz}{dt} = -\frac{\pi}{2};$$

$$C_3: \begin{cases} x=t \\ y=1-t \\ z=0 \end{cases} \text{ for } 0 \leq t \leq 1,$$

$$\frac{dx}{dt} = 1, \quad \frac{dy}{dt} = -1, \quad \frac{dz}{dt} = 0;$$

$$\text{Circ} = \int_C \vec{F} \cdot \vec{T} ds$$

$$= \int_{C_1} \vec{F} \cdot \vec{T} ds + \int_{C_2} \vec{F} \cdot \vec{T} ds + \int_{C_3} \vec{F} \cdot \vec{T} ds;$$

$$\int_{C_1} \vec{F} \cdot \vec{T} ds = \int_{C_1} M dx + N dy + P dz$$

$$= \int_0^{\frac{\pi}{2}} \left[(2x) \frac{dx}{dt} + (2z) \frac{dy}{dt} + (2y) \frac{dz}{dt} \right] dt$$

$$= \int_0^{\frac{\pi}{2}} [2 \cos t (\sin t) + (2t) \cos t + (2 \sin t)(1)] dt$$

$$= (-\sin^2 t + 2(t \sin t + \cos t) - 2 \cos t) \Big|_0^{\frac{\pi}{2}}$$

$$= (-\sin^2 t + 2t \sin t) \Big|_0^{\frac{\pi}{2}}$$

$$= \left(-\sin^2 \frac{\pi}{2} + 2\left(\frac{\pi}{2}\right) \sin \frac{\pi}{2} \right)$$

$$- (-\sin^2 0 + 0) = -(1)^2 + \pi(1) = \pi - 1;$$

$$\int_{C_2} \vec{F} \cdot \vec{T} ds = \int_{C_2} M dx + N dy + P dz$$

$$= \int_0^1 \left[(2x) \frac{dx}{dt} + (2z) \frac{dy}{dt} + (2y) \frac{dz}{dt} \right] dt$$

$$= \int_0^1 \left[(0)(0) + \pi(1-t)(0) + (2)\left(-\frac{\pi}{2}\right) \right] dt$$

$$= -\pi t \Big|_0^1 = -\pi;$$

$$\begin{aligned}
 \int_{C_3} \vec{F} \cdot \vec{T} \, ds &= \int_{C_3} M \, dx + N \, dy + P \, dz \\
 &= \int_0^1 \left[(2x) \frac{dx}{dt} + (2z) \frac{dy}{dt} + (2y) \frac{dz}{dt} \right] dt \\
 &= \int_0^1 \left[(2t)(1) + (0)(-1) + (2-2t)(0) \right] dt \\
 &= \int_0^1 2t \, dt = t^2 \Big|_0^1 = 1 \quad ; \quad \text{so} \\
 \text{Flow} &= (\pi - 1) + (-\pi) + (1) = 0.
 \end{aligned}$$

$$53) \quad \vec{F}(x, y, z) = (xy) \vec{i} + (y) \vec{j} + (-yz) \vec{k},$$

$$C: \begin{cases} x = t \\ y = t^2 \\ z = t \end{cases} \text{ for } 0 \leq t \leq 1; \quad \frac{dx}{dt} = 1, \frac{dy}{dt} = 2t, \frac{dz}{dt} = 1;$$

$$\text{Flow} = \int_C \vec{F} \cdot \vec{T} \, ds = \int_C M \, dx + N \, dy + P \, dz$$

$$= \int_0^1 \left[(xy) \frac{dx}{dt} + (y) \frac{dy}{dt} + (-yz) \frac{dz}{dt} \right] dt$$

$$= \int_0^1 \left[\cancel{(t^3)}(1) + (t^2)(2t) + \cancel{(-t^3)}(1) \right] dt$$

$$= \int_0^1 2t^3 \, dt = \frac{1}{2} t^4 \Big|_0^1 = \frac{1}{2}.$$