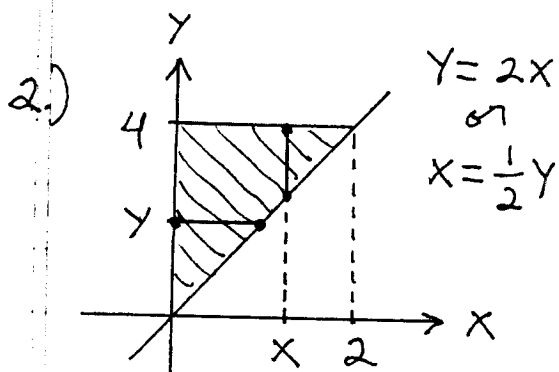
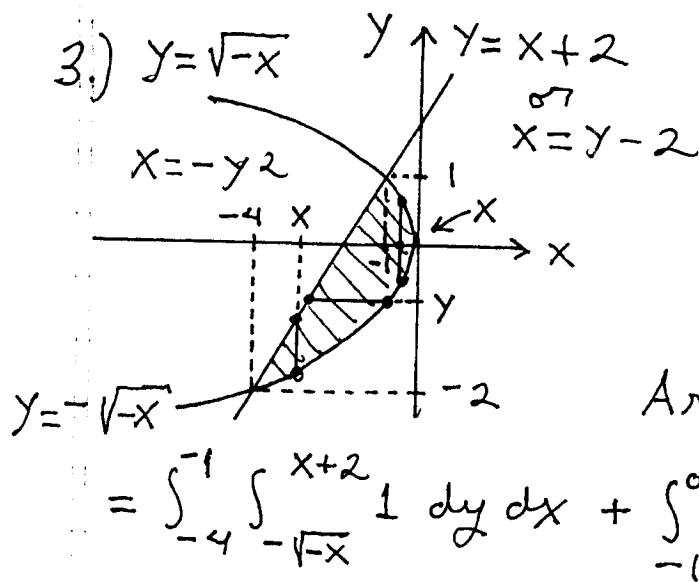


Section 15.3



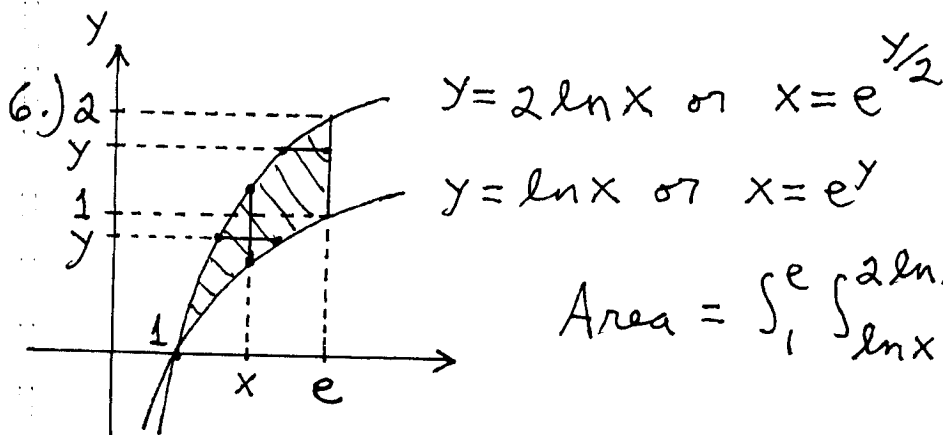
$$\begin{aligned} \text{Area} &= \int_0^2 \int_{2x}^4 1 \, dy \, dx \\ &= \int_0^4 \int_0^{\frac{1}{2}y} 1 \, dx \, dy \end{aligned}$$



$$\begin{aligned} -y^2 &= y - 2 \rightarrow \\ 0 &= y^2 + y - 2 \rightarrow \\ 0 &= (y - 1)(y + 2) \\ &\quad \downarrow \quad \downarrow \\ &\quad y = 1 \quad y = -2 \end{aligned}$$

$$\text{Area} = \int_{-2}^1 \int_{y-2}^{-y^2} 1 \, dx \, dy$$

$$= \int_{-4}^{-1} \int_{-\sqrt{-x}}^{x+2} 1 \, dy \, dx + \int_{-1}^0 \int_{-\sqrt{-x}}^{\sqrt{-x}} 1 \, dy \, dx$$



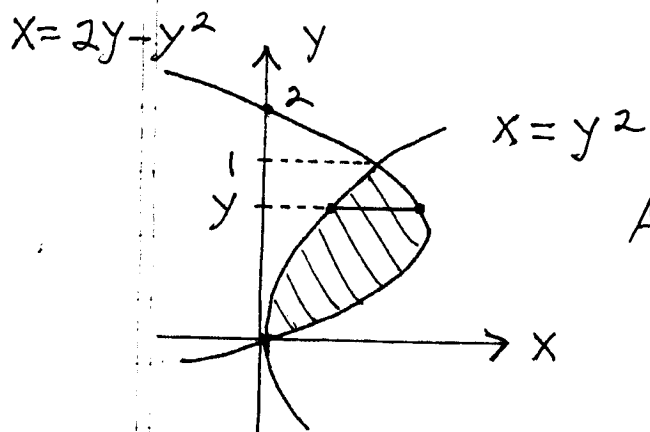
$$\text{Area} = \int_1^e \int_{\ln x}^{2 \ln x} 1 \, dy \, dx$$

$$= \int_0^1 \int_{e^{y/2}}^{e^y} 1 \, dx \, dy + \int_1^2 \int_{e^{y/2}}^e 1 \, dx \, dy$$

$$7.) y^2 = 2y - y^2 \rightarrow 2y^2 - 2y = 0 \rightarrow 2y(y-1) = 0$$

$$\downarrow \quad \downarrow$$

$$y=0 \quad y=1$$

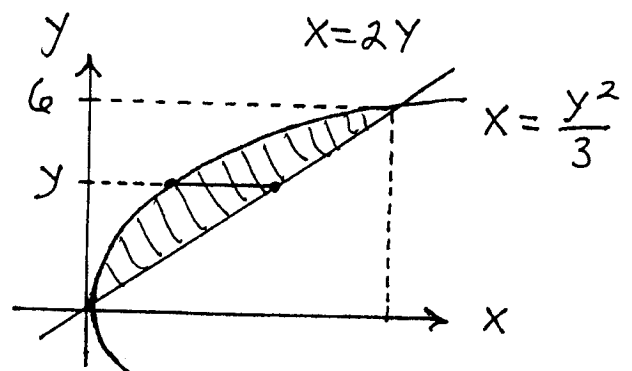


$$\text{Area} = \int_0^1 \int_{y^2}^{2y-y^2} 1 \, dx \, dy$$

$$13.) \int_0^6 \int_{y^2/3}^{2y} 1 \, dx \, dy$$

$$0 \leq y \leq 6$$

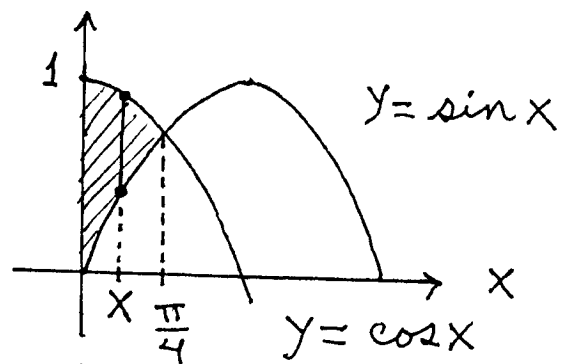
$$\frac{y^2}{3} \leq x \leq 2y$$



$$15.) \int_0^{\pi/4} \int_{\sin x}^{\cos x} 1 \, dy \, dx$$

$$0 \leq x \leq \frac{\pi}{4}$$

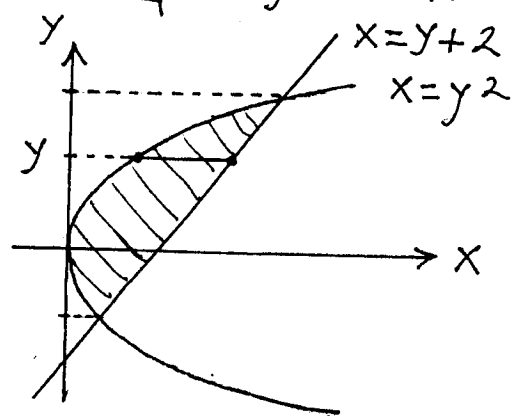
$$\sin x \leq y \leq \cos x$$



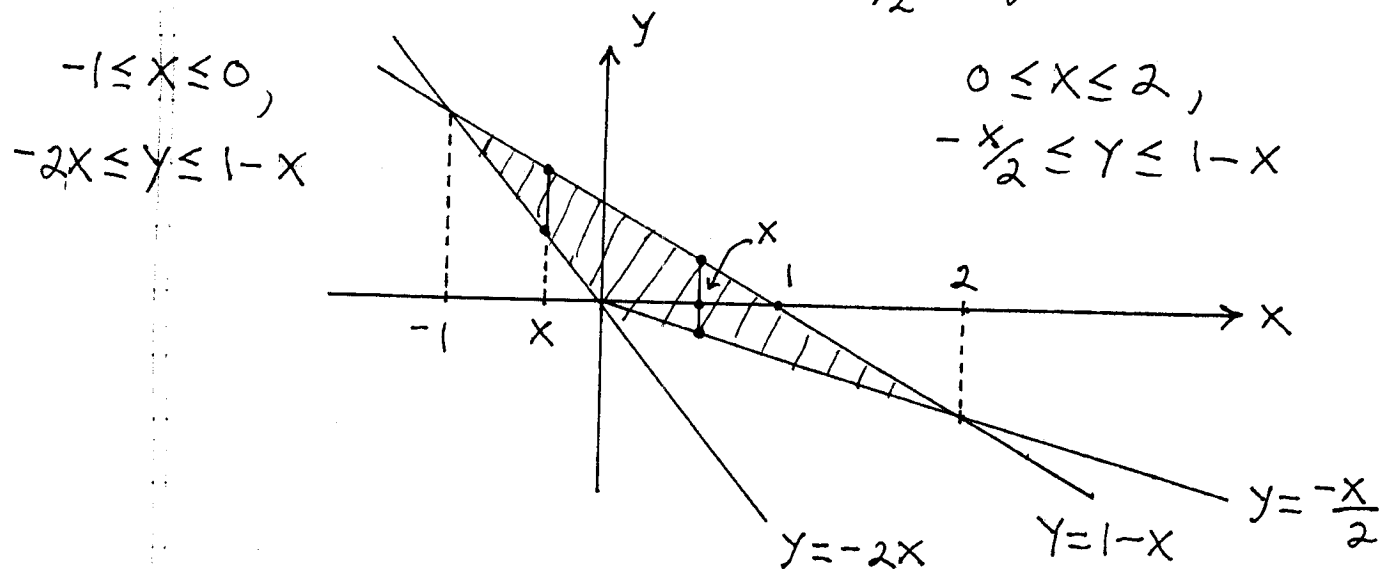
$$16.) \int_{-1}^2 \int_{y^2}^{y+2} 1 \, dx \, dy$$

$$-1 \leq y \leq 2$$

$$y^2 \leq x \leq y+2$$



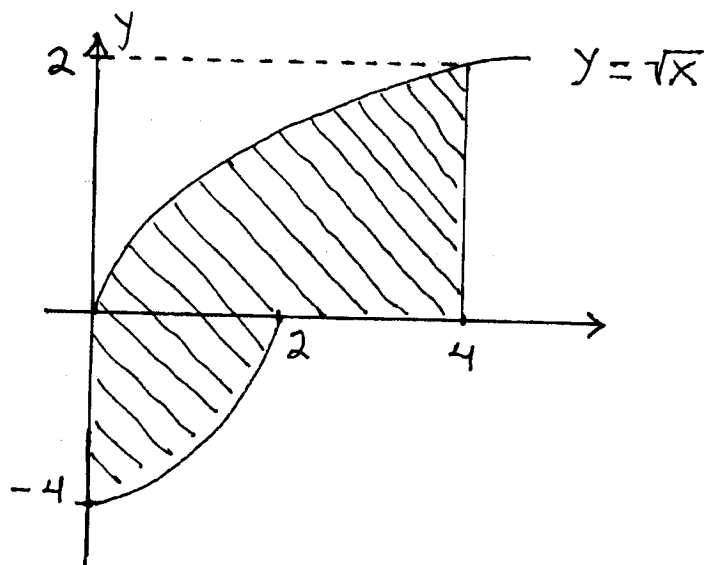
$$17.) \int_{-1}^0 \int_{-2x}^{1-x} 1 \, dy \, dx + \int_0^2 \int_{-\frac{x}{2}}^{1-x} 1 \, dy \, dx$$



$$18.) \int_0^2 \int_{x^2-4}^0 1 \, dy \, dx + \int_0^4 \int_0^{\sqrt{x}} 1 \, dy \, dx$$

$0 \leq x \leq 2,$
 $x^2 - 4 \leq y \leq 0$

$0 \leq x \leq 4,$
 $0 \leq y \leq \sqrt{x}$



$$19a) \text{ Ave} = \frac{1}{\text{area } R} \iint_R f(x,y) dA$$

$$= \frac{1}{\pi^2} \int_0^\pi \int_0^\pi \sin(x+y) dy dx$$

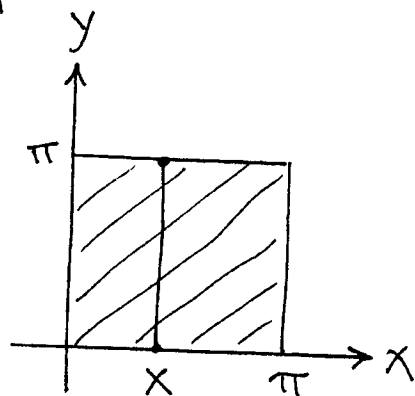
$$= \frac{1}{\pi^2} \int_0^\pi \left(-\cos(x+y) \Big|_{y=0}^{y=\pi} \right) dx$$

$$= \frac{1}{\pi^2} \int_0^\pi [-\cos(x+\pi) - -\cos x] dx$$

$$= \frac{1}{\pi^2} (-\sin(x+\pi) + \sin x) \Big|_0^\pi$$

$$= \frac{1}{\pi^2} (-\overset{0}{\cancel{\sin 2\pi}} + \overset{0}{\cancel{\sin \pi}}) - \frac{1}{\pi^2} (-\overset{0}{\cancel{\sin \pi}} + \overset{0}{\cancel{\sin 0}})$$

$$= 0$$



$$21) \text{ Ave} = \frac{1}{\text{area } R} \iint_R f(x,y) dA$$

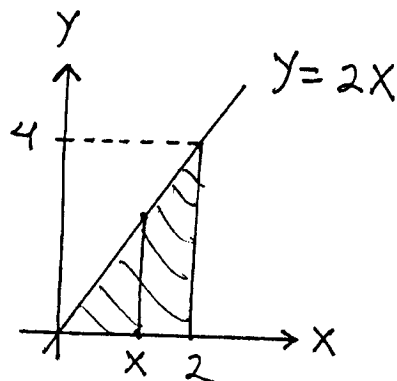
$$= \frac{1}{\frac{1}{2}(2)(4)} \int_0^2 \int_0^{2x} (x^2 + y^2) dy dx$$

$$= \frac{1}{4} \int_0^2 \left(x^2 y + \frac{1}{3} y^3 \right) \Big|_{y=0}^{y=2x} dx$$

$$= \frac{1}{4} \int_0^2 \left[(2x^3 + \frac{8}{3} x^3) - (0+0) \right] dx$$

$$= \frac{1}{4} \int_0^2 \frac{14}{3} x^3 dx = \frac{7}{6} \cdot \frac{1}{4} x^4 \Big|_0^2$$

$$= \frac{7}{24} (16) = \frac{14}{3}$$



$$22.) \text{Ave} = \frac{1}{\text{area } R} \iint_R f(x,y) dy dx;$$

$$\text{area } R = \int_0^1 \int_{e^x}^e 1 dy dx$$

$$= \int_0^1 (y \big|_{y=e^x}^{y=e}) dx$$

$$= \int_0^1 (e - e^x) dx = (ex - e^x) \big|_0^1$$

$$= (e - e) - (0 - 1) = 1;$$

$$\text{Ave} = \frac{1}{1} \int_0^1 \int_{e^x}^e \frac{1}{(x+1)y} dy dx$$

$$= \int_0^1 \left(\frac{1}{x+1} \cdot \ln y \big|_{y=e^x}^{y=e} \right) dx$$

$$= \int_0^1 \left[\frac{1}{x+1} \cdot \ln e - \frac{1}{x+1} \cdot \ln e^x \right] dx$$

$$= \int_0^1 \left[\frac{1}{x+1} - \frac{x}{x+1} \right] dx = \int_0^1 \frac{1-x}{x+1} dx$$

(Let $u = x+1 \xrightarrow{D} du = dx$ and $x = u-1$,
and $x: 0 \rightarrow 1$ so $u: 1 \rightarrow 2$)

$$= \int_1^2 \frac{1-(u-1)}{u} du = \int_1^2 \frac{2-u}{u} du$$

$$= \int_1^2 \left(\frac{2}{u} - 1 \right) du = (2 \ln |u| - u) \big|_1^2$$

$$= (2 \ln 2 - 2) - (2 \ln 1 - 1)$$

$$= 2 \ln 2 - 1$$

