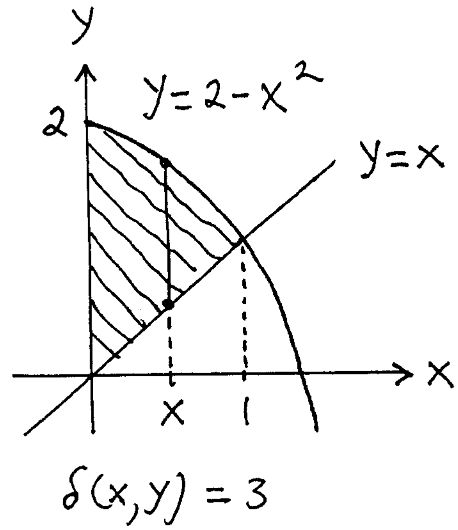
Section 15.6

1) $M = \iint_R \delta(x,y)\, dA$

$= \int_0^1 \int_x^{2-x^2} 3 \; dy\, dx$,



$M_y = \iint_R x\, \delta(x,y)\, dA$

$= \int_0^1 \int_x^{2-x^2} 3x \; dy\, dx$,

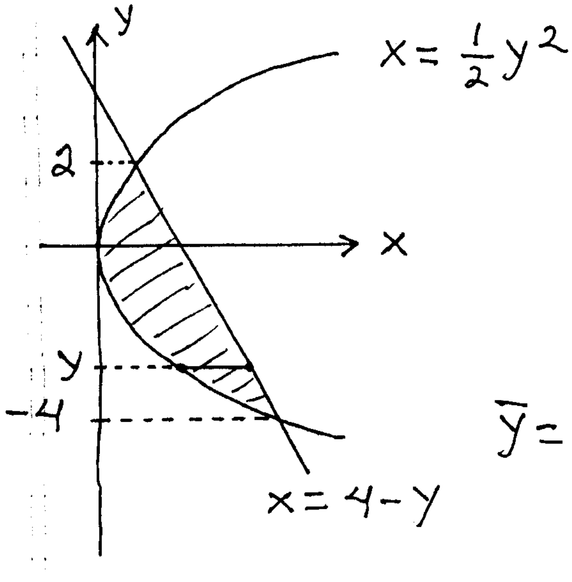
$M_x = \iint_R y\, \delta(x,y)\, dA = \int_0^1 \int_x^{2-x^2} 3y \; dy\, dx \rightarrow$

$\bar{x} = \frac{M_y}{M}$ and $\bar{y} = \frac{M_x}{M}$

3) $y^2 = 2x \rightarrow x = \frac{1}{2}y^2$ and $x + y = 4 \rightarrow$

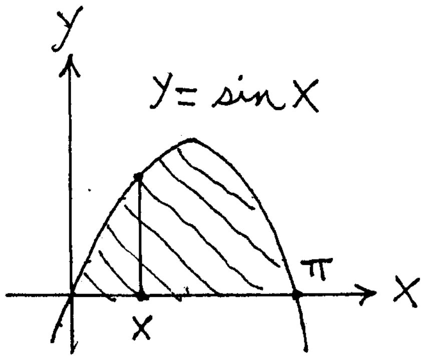
$\frac{1}{2}y^2 + y = 4 \rightarrow y^2 + 2y - 8 = 0 \rightarrow$

$(y-2)(y+4) = 0 \rightarrow y = 2, \; y = -4$



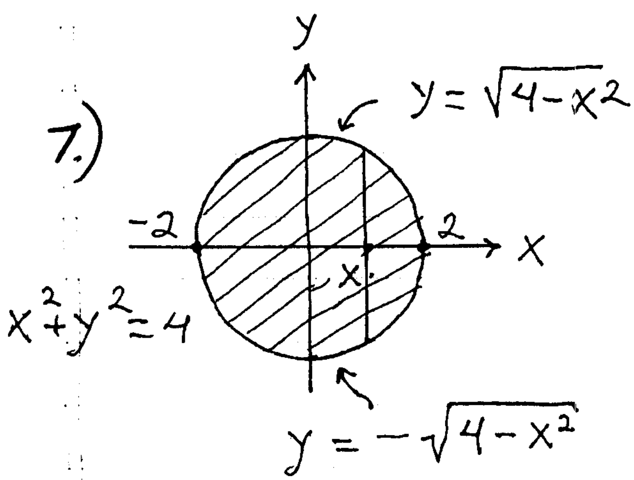
$$\bar{x} = \frac{\int_{-4}^{2} \int_{\frac{1}{2}y^2}^{4-y} x \; dx\, dy}{\int_{-4}^{2} \int_{\frac{1}{2}y^2}^{4-y} 1 \; dx\, dy} ,$$

$$\bar{y} = \frac{\int_{-4}^{2} \int_{\frac{1}{2}y^2}^{4-y} y \; dx\, dy}{\int_{-4}^{2} \int_{\frac{1}{2}y^2}^{4-y} 1 \; dx\, dy}$$

6) 

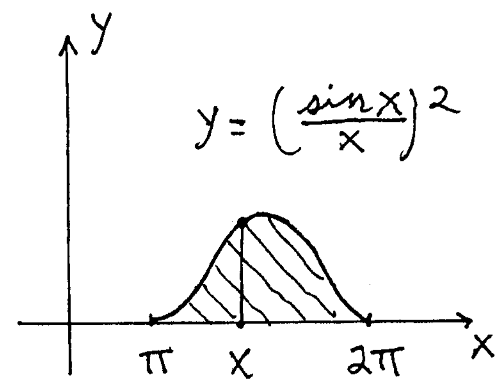
$$\overline{x} = \frac{\int_0^{\pi}\int_0^{\sin x} x \, dy\, dx}{\int_0^{\pi}\int_0^{\sin x} 1 \, dy\, dx},$$

$$\overline{y} = \frac{\int_0^{\pi}\int_0^{\sin x} y \, dy\, dx}{\int_0^{\pi}\int_0^{\sin x} 1 \, dy\, dx}$$

7) 

$$I_x = \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} y^2 \cdot (1)\, dy\, dx$$

8.) $I_y = \int_{\pi}^{2\pi} \int_0^{\left(\frac{\sin x}{x}\right)^2} x^2 (1)\, dy\, dx$



12.) $x^2 + 4y^2 = 12$ and $x = 4y^2$ →

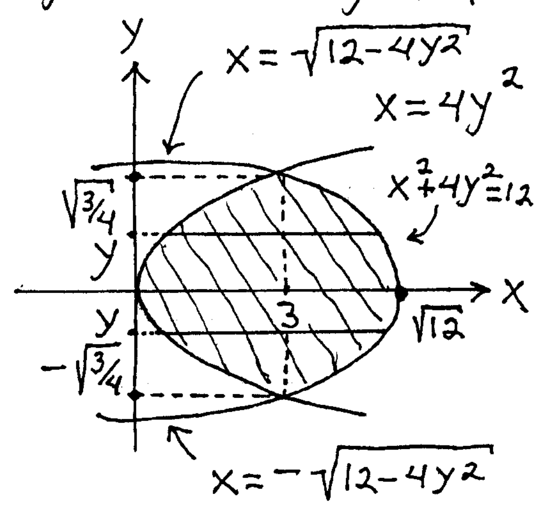
$x^2 + x - 12 = 0 \rightarrow (x-3)(x+4) = 0 \rightarrow x = 3, \cancel{x = -4}$

$\rightarrow 3^2 + 4y^2 = 12 \rightarrow y^2 = 3/4 \rightarrow$

$y = \pm\sqrt{3/4}$, $\delta(x,y) = 5x$,

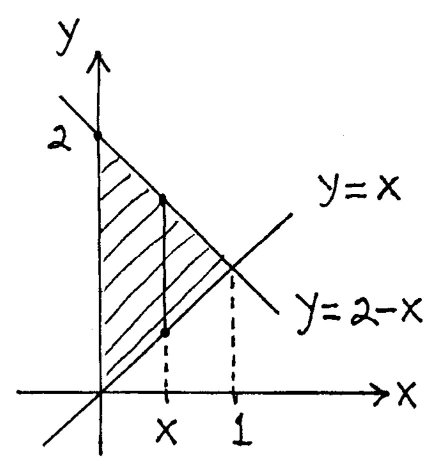
$$\text{Mass} = \int_0^{\sqrt{3/4}} \int_{4y^2}^{\sqrt{12-4y^2}} 5x \, dx\, dy$$

$$+ \int_{-\sqrt{3/4}}^{0} \int_{4y^2}^{-\sqrt{12-4y^2}} 5x \, dx\, dy$$



13.) $\delta(x,y) = 6x + 3y + 3$

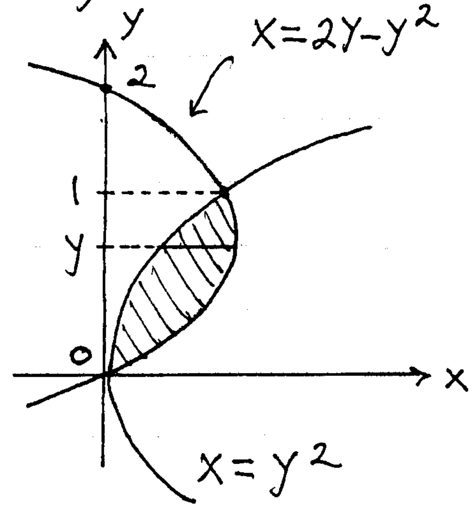
$$\bar{x} = \frac{\int_0^1 \int_x^{2-x} x \cdot (6x + 3y + 3)\, dy\, dx}{\int_0^1 \int_x^{2-x} (6x + 3y + 3)\, dy\, dx},$$



$$\bar{y} = \frac{\int_0^1 \int_x^{2-x} y\,(6x + 3y + 3)\, dy\, dx}{\int_0^1 \int_x^{2-x} (6x + 3y + 3)\, dy\, dx}$$

14.) $x = y^2$ and $x = 2y - y^2 \rightarrow y^2 = 2y - y^2 \rightarrow$
$2y^2 - 2y = 2y(y-1) = 0 \rightarrow y = 0,\ y = 1$

$\delta(x,y) = y+1$

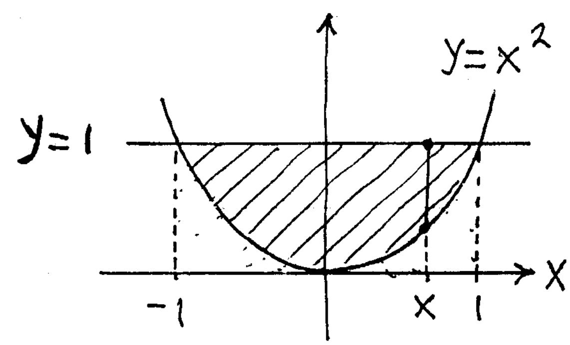


$$\bar{x} = \frac{\int_0^1 \int_{y^2}^{2y-y^2} x(y+1)\,dx\,dy}{\int_0^1 \int_{y^2}^{2y-y^2} (y+1)\,dx\,dy},$$

$$\bar{y} = \frac{\int_0^1 \int_{y^2}^{2y-y^2} y(y+1)\,dx\,dy}{\int_0^1 \int_{y^2}^{2y-y^2} (y+1)\,dx\,dy};$$

$$I_x = \int_0^1 \int_{y^2}^{2y-y^2} y^2(y+1)\,dx\,dy$$

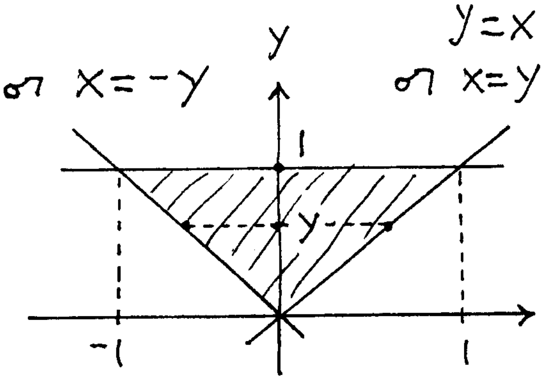
16.) $\delta(x,y) = y+1$



$$\bar{x} = \frac{\int_{-1}^1 \int_{x^2}^{1} x(y+1)\,dy\,dx}{\int_{-1}^1 \int_{x^2}^{1} (y+1)\,dy\,dx},$$

$$\bar{y} = \frac{\int_{-1}^1 \int_{x^2}^{1} y(y+1)\,dy\,dx}{\int_{-1}^1 \int_{x^2}^{1} (y+1)\,dy\,dx};$$

$$I_y = \int_{-1}^{1}\int_{x^2}^{1} x^2\,(y+1)\,dy\,dx$$



19.) $\delta(x,y) = y+1$

$$\bar{x} = \frac{\int_0^1\int_{-y}^{y} x\,(y+1)\,dx\,dy}{\int_0^1\int_{-y}^{y} (y+1)\,dx\,dy},$$

$$\bar{y} = \frac{\int_0^1\int_{-y}^{y} y\,(y+1)\,dx\,dy}{\int_0^1\int_{-y}^{y} (y+1)\,dx\,dy};$$

$$I_x = \int_0^1\int_{-y}^{y} y^2\,(y+1)\,dx\,dy,$$

$$I_y = \int_0^1\int_{-y}^{y} x^2\,(y+1)\,dx\,dy,$$

$$I_0 = \int_0^1\int_{-y}^{y} (x^2+y^2)\,(y+1)\,dx\,dy$$