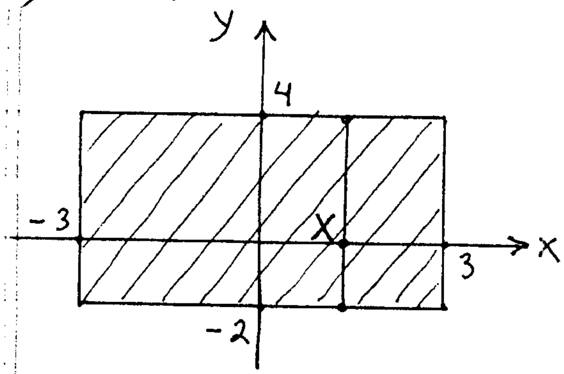
Section 15.6

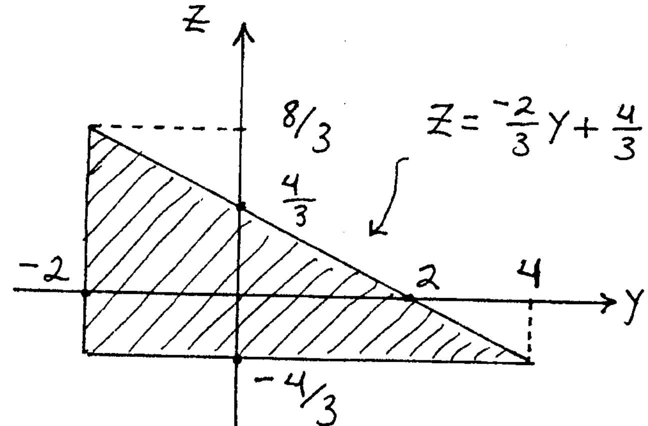
21) $I_x = \iiint_R r^2 \delta(P)\,dV = \int_0^a \int_0^b \int_0^c (y^2+z^2)\cdot(1)\,dz\,dy\,dx$,

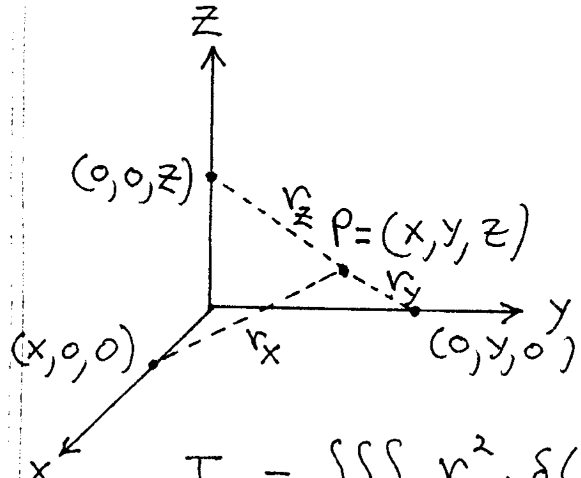
$I_0 = \int_0^a \int_0^b \int_0^c (x^2+y^2+z^2)(1)\,dz\,dy\,dx$

22) projection of R onto xy-plane:



projection onto yz-plane:





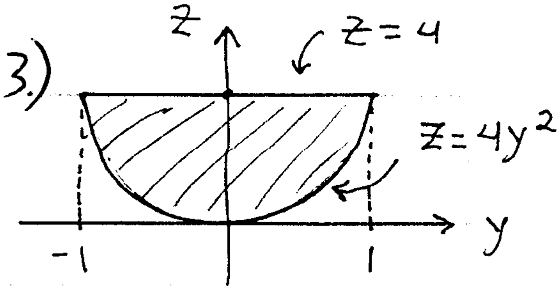
$\delta = 1$

$$I_x = \iiint_R r_x^2 \cdot \delta(P)\,dV = \int_{-3}^{3}\int_{-2}^{4}\int_{-\frac{4}{3}}^{-\frac{2}{3}y+\frac{4}{3}} (y^2+z^2)(1)\,dz\,dy\,dx ,$$

$$I_y = \iiint_R r_y^2 \cdot \delta(P)\,dV = \int_{-3}^{3}\int_{-2}^{4}\int_{-\frac{4}{3}}^{-\frac{2}{3}y+\frac{4}{3}} (x^2+z^2)(1)\,dz\,dy\,dx ,$$

$$I_z = \iiint_R r_z^2 \cdot \delta(P)\,dV = \int_{-3}^{3}\int_{-2}^{4}\int_{-\frac{4}{3}}^{-\frac{2}{3}y+\frac{4}{3}} (x^2+y^2)(1)\,dz\,dy\,dx$$

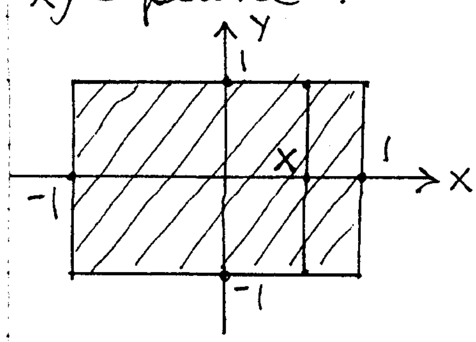
23)



projection onto yz-plane:

$\delta(P) = x + y + z + 2$

projection onto xy-plane:



moment of inertia about y-axis is

$$I_y = \int_{-1}^{1}\int_{-1}^{1}\int_{4y^2}^{4} (x^2+z^2)(x+y+z+2)\,dz\,dy\,dx \,;$$

center of mass is

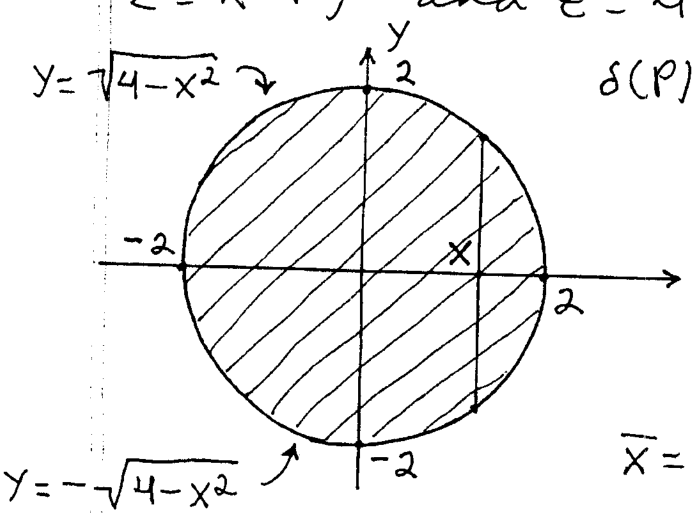
$$\bar{x} = \frac{\int_{-1}^{1}\int_{-1}^{1}\int_{4y^2}^{4} x\cdot(x+y+z+2)\,dz\,dy\,dx}{\int_{-1}^{1}\int_{-1}^{1}\int_{4y^2}^{4} (x+y+z+2)\,dz\,dy\,dx},$$

$$\bar{y} = \frac{\int_{-1}^{1}\int_{-1}^{1}\int_{4y^2}^{4} y\cdot(x+y+z+2)\,dz\,dy\,dx}{\int_{-1}^{1}\int_{-1}^{1}\int_{4y^2}^{4} (x+y+z+2)\,dz\,dy\,dx},$$

$$\bar{z} = \frac{\int_{-1}^{1}\int_{-1}^{1}\int_{4y^2}^{4} z\,(x+y+z+2)\,dz\,dy\,dx}{\int_{-1}^{1}\int_{-1}^{1}\int_{4y^2}^{4} (x+y+z+2)\,dz\,dy\,dx}.$$

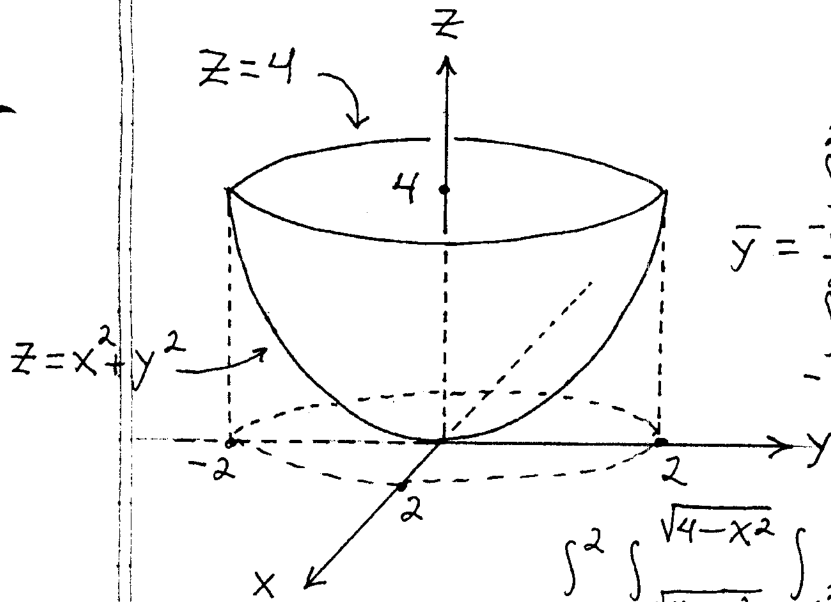
25) a.) projection onto XY-plane :

$z = x^2+y^2$ and $z = 4 \rightarrow x^2+y^2 = 4$



$\delta(P) = e^{-(x^2+y^2+z^2)}$, center of mass is

$$\bar{x} = \frac{\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+y^2}^{4} x\cdot e^{-(x^2+y^2+z^2)}\,dz\,dy\,dx}{\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+y^2}^{4} e^{-(x^2+y^2+z^2)}\,dz\,dy\,dx},$$



$$\bar{y} = \frac{\displaystyle\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+y^2}^{4} y e^{-(x^2+y^2+z^2)}\,dz\,dy\,dx}{\displaystyle\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+y^2}^{4} e^{-(x^2+y^2+z^2)}\,dz\,dy\,dx},$$

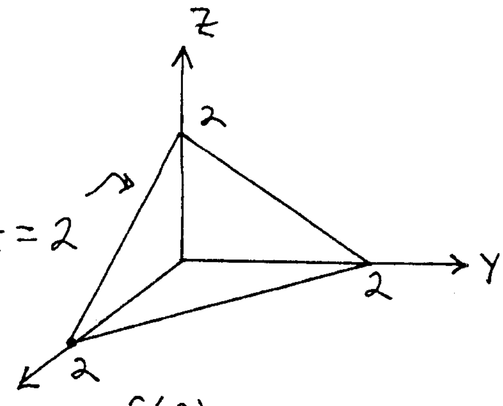
$$\bar{z} = \frac{\displaystyle\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+y^2}^{4} z e^{-(x^2+y^2+z^2)}\,dz\,dy\,dx}{\displaystyle\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+y^2}^{4} e^{-(x^2+y^2+z^2)}\,dz\,dy\,dx}$$

29.)

$x+y+z=2$

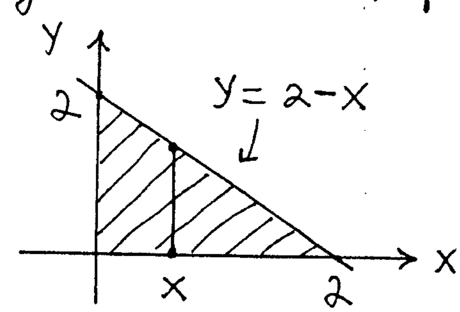
or

$z=2-x-y$



$\delta(P) = 2x$

projection onto xy-plane:



a.) Mass $= \iiint_R \delta(P)\, dV = \int_0^2 \int_0^{2-x} \int_0^{2-x-y} 2x \, dz\, dy\, dx$

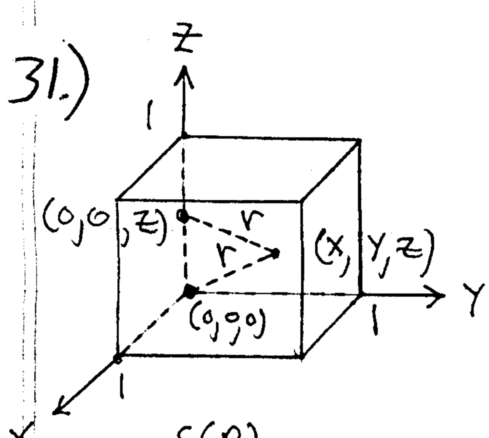
b.) center of mass is

$$\bar{x} = \frac{\int_0^2 \int_0^{2-x} \int_0^{2-x-y} x\cdot(2x)\, dz\, dy\, dx}{\int_0^2 \int_0^{2-x} \int_0^{2-x-y} 2x\, dz\, dy\, dx},$$

$$\bar{y} = \frac{\int_0^2 \int_0^{2-x} \int_0^{2-x-y} y\cdot(2x)\, dz\, dy\, dx}{\int_0^2 \int_0^{2-x} \int_0^{2-x-y} 2x\, dz\, dy\, dx},$$

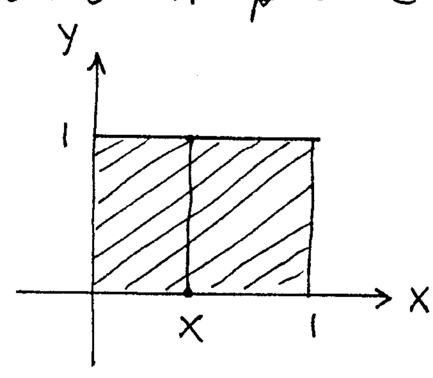
$$\bar{z} = \frac{\int_0^2 \int_0^{2-x} \int_0^{2-x-y} z\cdot(2x)\, dz\, dy\, dx}{\int_0^2 \int_0^{2-x} \int_0^{2-x-y} 2x\, dz\, dy\, dx}$$

31.)



$\delta(P) = x+y+z+1$

projection onto xy-plane:



a.) Mass $= \int_0^1\int_0^1\int_0^1 (x+y+z+1)\,dz\,dy\,dx$

b.) center of mass is

$$\bar{x} = \frac{\int_0^1\int_0^1\int_0^1 x(x+y+z+1)\,dz\,dy\,dx}{\int_0^1\int_0^1\int_0^1 (x+y+z+1)\,dz\,dy\,dx},$$

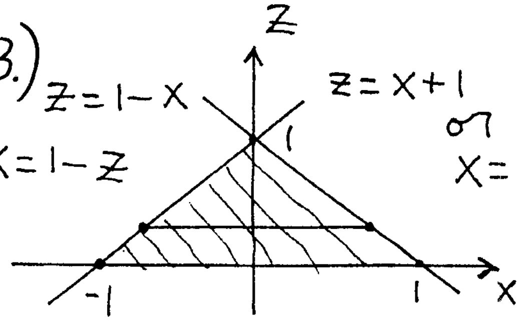
$$\bar{y} = \frac{\int_0^1\int_0^1\int_0^1 y(x+y+z+1)\,dz\,dy\,dx}{\int_0^1\int_0^1\int_0^1 (x+y+z+1)\,dz\,dy\,dx},$$

$$\bar{z} = \frac{\int_0^1\int_0^1\int_0^1 z(x+y+z+1)\,dz\,dy\,dx}{\int_0^1\int_0^1\int_0^1 (x+y+z+1)\,dz\,dy\,dx}$$

c.) $I_z = \int_0^1\int_0^1\int_0^1 (x^2+y^2)(x+y+z+1)\,dz\,dy\,dx$,

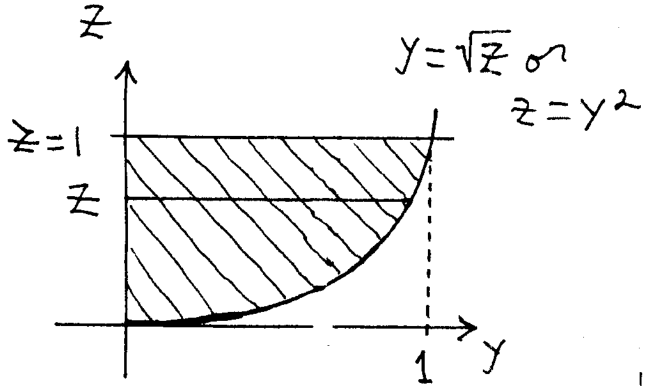
$I_0 = \int_0^1\int_0^1\int_0^1 (x^2+y^2+z^2)(x+y+z+1)\,dz\,dy\,dx$

33.) $z=1-x$ or $x=1-z$



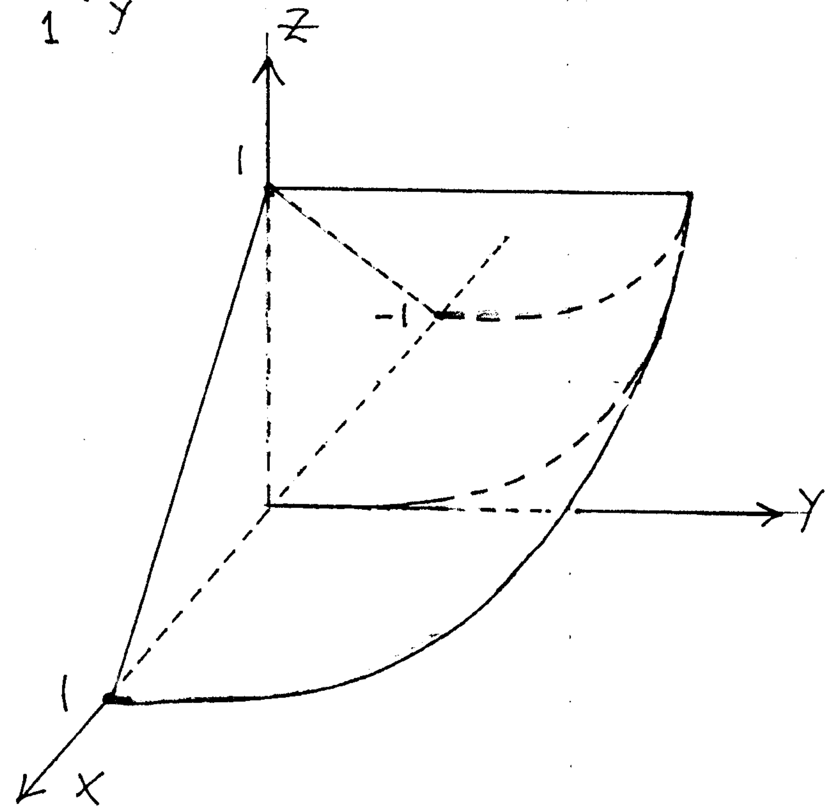
$z=x+1$ or $x=z-1$

projection onto xz-plane:



projection onto yz-plane :

$\delta(P) = 2y + 5$



$\delta(P) = 2y + 5$

$0 \le z \le 1$,
$1 - z \le x \le z - 1$,
$0 \le y \le \sqrt{z}$

$$\text{Mass} = \int_0^1 \int_{1-z}^{z-1} \int_0^{\sqrt{z}} (2y+5)\, dy\, dx\, dz$$