Math 21B | Midterm 1 | Spring 2009

NAME(print in CAPITAL letters, first name first): Key

NAME(sign): ________________

ID#: ________________

Instructions: There are four problems. Some questions are easier than others so you are encouraged to read the entire exam before beginning your work. Make sure that you have all 4 problems.

1	
2	
3	
4	
TOTAL	

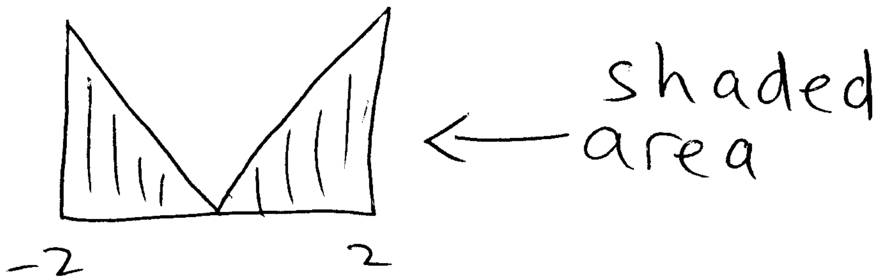
$$\begin{aligned}
\sin A \sin B &= \frac{1}{2}(\cos(A-B) - \cos(A+B)) \\
\sin A \cos B &= \frac{1}{2}(\sin(A-B) + \sin(A+B)) \\
\cos A \cos B &= \frac{1}{2}(\cos(A-B) + \cos(A+B))
\end{aligned}$$

$$\sin^2 A = \frac{1}{2}(1 - \cos(2A)), \quad \cos^2 A = \frac{1}{2}(1 + \cos(2A))$$

1. Multiple choice (5 points each). Circle the correct answer.

   (a) Find $\int_{-2}^{2} |x|\, dx$.

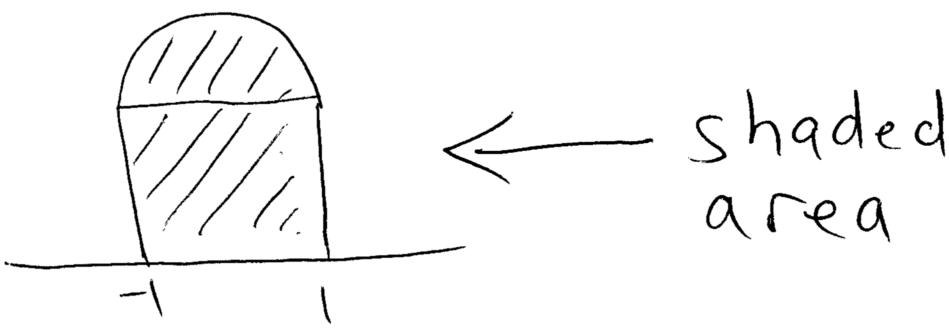
   $0 \qquad 1 \qquad 2/3 \qquad 4/3 \qquad 4 \qquad$ none of the above



   (b) Evaluate $\int_{-1}^{1} 2 + \sqrt{1-x^2}\, dx$.

   $0 \qquad 1 \qquad 2 \qquad 4 \qquad \pi \qquad 2+2\pi$

   $4+4\pi \qquad 4+\pi \qquad 4+\pi/2 \qquad$ none of the above



even

(c) Evaluate $\int_{-1}^{1} x^2 + |x|\, dx$.

0 1 2/3 4/3 5/3 8/3 2 3

none of the above

$$2 \int_0^1 x^2 + x\, dx$$

$$= 2 \left[\frac{1}{3}x^3 + \frac{1}{2}x^2 \right]_0^1$$

$$= 2\left(\frac{1}{3} + \frac{1}{2}\right) = \frac{5}{3}$$

(d) Evaluate $\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} e^{k/n}$.

e $e-1$ $e-2$ 0 1 e^2

e^2-1 e^2-2 none of the above

$$\sum_{k=1}^{n} e^{k/n} \frac{1}{n}$$

$$= \sum_{k=1}^{n} f(k/n) \frac{1}{n}, \quad f(x) = e^x$$

$$\to \int_0^1 e^x\, dx = [e^x]_0^1$$

$$= e - 1$$

2. (20 points.)

(a) Evaluate $\int_{-1}^{1} 4 + e^{x^2} \sin(2\pi x)\, dx$.
(Hint: split the integral into two parts.)

$$\int_{-1}^{1} 4\, dx + \int_{-1}^{1} \underbrace{e^{x^2} \sin(2\pi x)}_{\text{odd}}\, dx$$

$$= 8$$

(b) Let A be the answer to part (a). Which of the quantities below is the closest approximation to $\int_{-1}^{1.01} 4 + e^{x^2} \sin(2\pi x)\, dx$? (Please circle.)

A $\quad A+1 \quad A+2 \quad A+e^{1.01} \quad A+e^{2.02}$

$A+\pi \quad A+2\pi \quad A+0.01 \quad A+0.02$

$A+2.03 \quad A+0.03 \quad A+0.04$

Let $F(y) = \int_{-1}^{y} 4 + e^{x^2} \sin(2\pi x)\, dx$

$F'(y) = 4 + e^{y^2} \sin(2\pi y)$

$F'(1) = 4$

$F(1+0.01) \approx F(1) + F'(1) \cdot 0.01$

$= A + 4 \cdot 0.01 = A + 0.04$

3. (10 points.) Let $f(x) = \int_0^x \ln(1-t^3)\,dt$, $-\frac{1}{2} \le x \le \frac{1}{2}$.

What value of x in $[-\frac{1}{2}, \frac{1}{2}]$ maximizes $f(x)$?

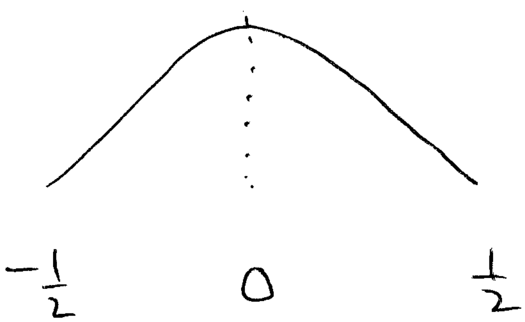
$f'(x) = \ln(1-x^3)$,

which is

> 0 if $x < 0$

$= 0$ if $x = 0$

< 0 if $x > 0$



So the max is at $x = 0$

4. (40 points.) Evaluate the following integrals.

(a) $\int x\sqrt{4-x^2}\,dx$

$u = 4 - x^2$

$du = -2x\,dx$

$$= -\frac{1}{2}\int \sqrt{u}\,du$$

$$= -\frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C$$

$$= -\frac{1}{3}(4-x^2)^{3/2} + C$$

(b) $\int_2^4 \frac{1}{x\ln x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$\int_{\ln 2}^{\ln 4}\frac{1}{u}\,du$$

$$= \Big[\ln|u|\Big]_{\ln 2}^{\ln 4}$$

$\ln 2^2 = 2\ln 2$

$$= \ln\ln 4 - \ln\ln 2 \quad = \ln\frac{\ln 4}{\ln 2}$$

$$= \boxed{\ln 2}$$

(c) $\int \frac{1}{\sqrt{e^{2x}-1}}\,dx$

$$= \int \frac{e^{-x}}{e^{-x}\sqrt{e^{2x}-1}}\,dx$$

$$= \int \frac{e^{-x}\,dx}{\sqrt{1-e^{-2x}}}$$

$u = e^{-x}$
$du = -e^{-x}dx$

$$= -\int \frac{du}{\sqrt{1-u^2}}$$

$$= -\sin^{-1}(u) + c = -\sin^{-1}(e^{-x}) + C$$

(d) $\int x^2\sqrt{x+1}\,dx$

$u = x+1$, $x = u-1$
$du = dx$

$$= \int (u-1)^2\sqrt{u}\,du$$

$$= \int (u^2-2u+1)\sqrt{u}\,du$$

$$= \int u^{5/2} - 2u^{3/2} + u^{1/2}\,du$$

$$= \frac{2}{7}u^{7/2} - \frac{4}{5}u^{5/2} + \frac{2}{3}u^{3/2} + c$$

$$= \frac{2}{7}(x+1)^{7/2} - \frac{4}{5}(x+1)^{5/2} + \frac{2}{3}(x+1)^{3/2} + C$$