Math 21B **Midterm 1** **Fall 2012**

NAME(print in CAPITAL letters, first name first): Key

NAME(sign): ______________________

ID#: ______________________

Instructions: There are five problems. Some questions are easier than others so you are encouraged to read the entire exam before beginning your work. Make sure that you have all 5 problems.

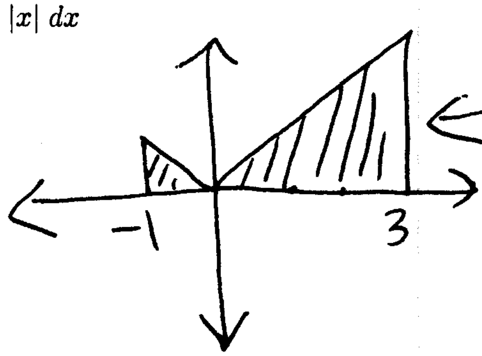
Points received:

1	
2	
3	
4	
5	
TOTAL	

1. Evaluate the following definite integrals.

(a) $\int_{-1}^{3} |x| \, dx$

$$\int_{-1}^{3} |x| \, dx =$$



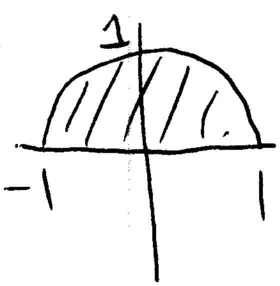
$$\leftarrow \text{area} =$$

$$\frac{1}{2}\cdot 1^2 + \frac{1}{2}\cdot 3^2 = 5$$

(b) $\int_{-1}^{1} 2\sqrt{1-x^2} \, dx$

$$= 2 \int_{-1}^{1} \sqrt{1-x^2} \, dx$$

$$= 2 \text{ area}\Big($$



$$\Big)$$

$$= \pi$$

3. Find $\lim_{x \to 0} \frac{1}{x^2} \int_0^x \frac{t}{1+\sin t}\, dt$.

$$\lim_{x \to 0} \frac{\int_0^x \frac{t}{1+\sin t}\, dt}{x^2} =$$

L'hopital

$$\lim_{x \to 0} \frac{\frac{d}{dx}\left[\int_0^x \frac{t}{1+\sin t}\, dt\right]}{\frac{d}{dx}\left[x^2\right]} =$$

$$\lim_{x \to 0} \frac{\left(\frac{x}{1+\sin x}\right)}{2x} =$$

$$\lim_{x \to 0} \frac{1}{2(1+\sin x)} =$$

$$\frac{1}{2(1+\sin 0)} = \frac{1}{2}$$

2. Solve the initial value problem

$$\frac{d^2s}{dt^2} = e^{-t}, \quad s'(0) = 0, \quad s(0) = 0 .$$

$$s'(t) = -e^{-t} + C$$

$$0 = s'(0) = -e^{0} + C$$

$$= -1 + C$$

$$\Rightarrow C = 1$$

$$s'(t) = -e^{-t} + 1$$

$$s(t) = e^{-t} + t + B$$

$$0 = s(0) = e^{-0} + 0 + B$$

$$= 1 + B$$

$$\Rightarrow B = -1$$

$$\Rightarrow s(t) = e^{-t} + t - 1$$

4. Evaluate the following indefinite integrals.

(a) $\int \sqrt{\dfrac{x^2-1}{x^8}}\,dx$

$$= \int \frac{1}{x^3}\sqrt{\frac{x^2-1}{x^2}}\,dx$$

$$= \int \frac{1}{x^3}\sqrt{1-\frac{1}{x^2}}\,dx$$

$$\left[\begin{array}{l} u = 1-\dfrac{1}{x^2} \\ du = 2x^{-3}\,dx \end{array}\right.$$

$$= \frac{1}{2}\int \sqrt{u}\,du$$

$$= \frac{1}{2}\,\frac{u^{3/2}}{3/2} + C = \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} + C$$

(b) $\int \dfrac{\ln x}{x}\,dx$

$$\left[\begin{array}{l} u = \ln x \\ du = \dfrac{1}{x}\,dx \end{array}\right.$$

$$= \int u\,du$$

$$= \frac{1}{2}u^2 + C \quad = \frac{1}{2}(\ln x)^2 + C$$

5. Write the limit

$$\lim_{n\to\infty}\left[\sum_{k=1}^{n}\left(1+\frac{2k}{n}\right)^4\frac{2}{n}\right]$$

as a definite integral and evaluate it.

$f(x) = x^4$

Let $P = \{1, 1+\frac{2}{n}, 1+\frac{4}{n}, \ldots, \underbrace{1+\frac{2n}{n}}_{3}\}$

Then P is a partition of $[1,3]$. $\Delta x = \frac{2}{n}$

$$\lim_{n\to\infty}\left[\sum_{k=1}^{n}\left(1+\frac{2k}{n}\right)^4\frac{2}{n}\right] =$$

$$\lim_{\|P\|\to 0}\sum_{k=1}^{n} f(c_k)\,\Delta x =$$

$$\int_1^3 x^4\,dx =$$

$$\frac{1}{5}x^5\Big]_1^3 = \frac{1}{5}3^5 - \frac{1}{5}1^5$$

$$= \frac{242}{5}$$