Math 21B Midterm 2 Fall 2012

NAME(print in CAPITAL letters, first name first): KEY

NAME(sign): _____

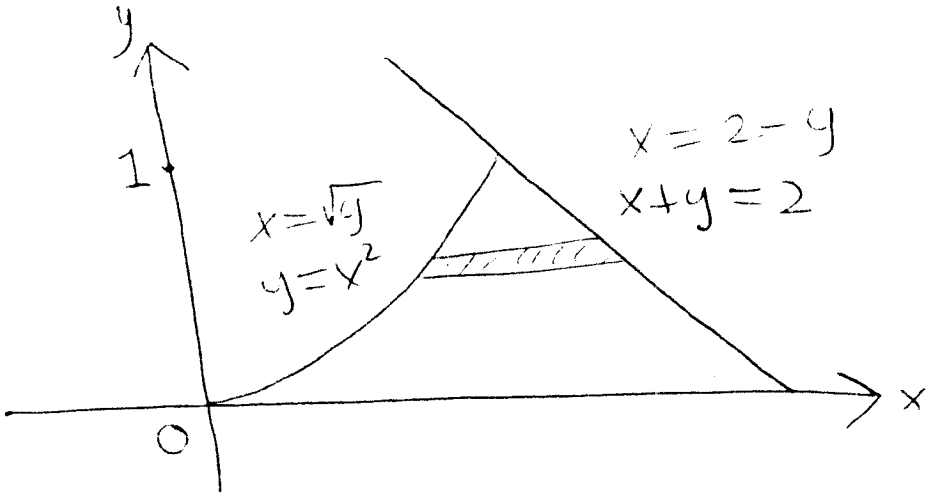
ID#: _____

Instructions: There are five problems. Some questions are easier than others so you are encouraged to read the entire exam before beginning your work. Make sure that you have all 5 problems.

Points received:

1	
2	
3	
4	
5	
TOTAL	

1. (20 points.) Find the area of the region bounded by the graphs of $y = 0$, $y = x^2$, and $x + y = 2$.



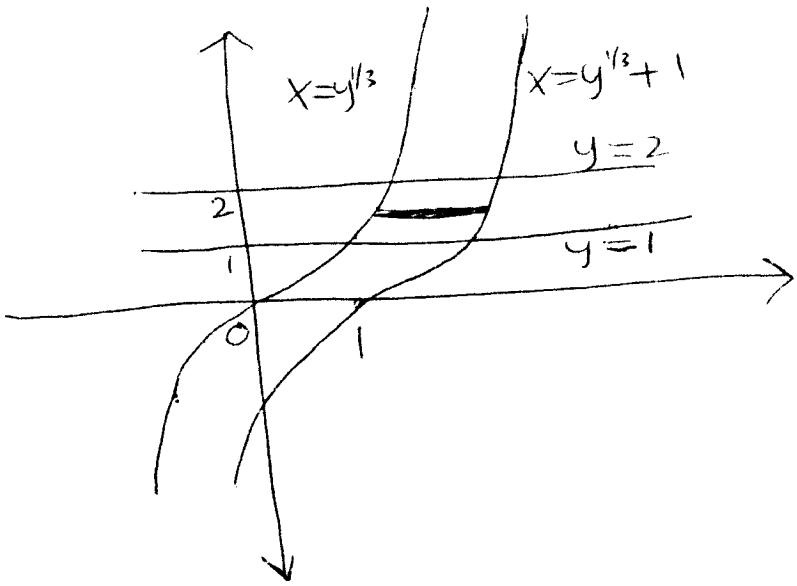
$$A = \int_0^1 2 - y - \sqrt{y}\, dy$$

$$= 2y - \frac{1}{2}y^2 - \frac{2}{3}y^{3/2} \Big]_0^1$$

$$= \left(2 - \frac{1}{2} - \frac{2}{3}\right) - 0$$

$$= 5/6$$

2. (20 points.) Find the area of the region bounded by the graphs of $y = x^3$, $y = (x-1)^3$, $y = 1$ and $y = 2$.



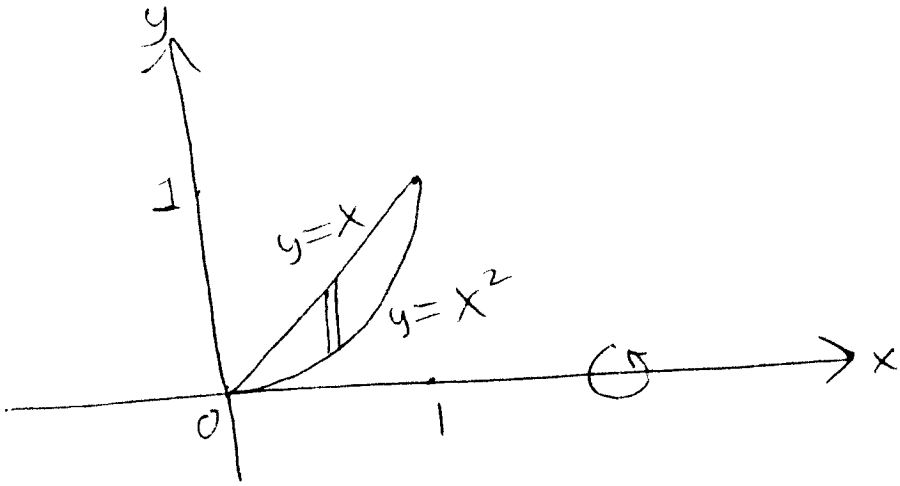
$$A = \int_1^2 (y^{1/3}+1) - y^{1/3}\, dy$$

$$= \int_1^2 1\, dy$$

$$= y\Big]_1^2$$

$$= 2-1 = 1$$

3. (20 points.) The region bounded by the graphs of $y = x$ and $y = x^2$ is revolved about the x-axis. Find the volume of the resulting solid.



$$V = \pi \int_0^1 x^2 - (x^2)^2 \, dx$$

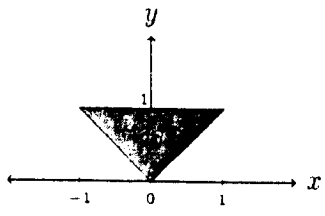
$$= \pi \int_0^1 x^2 - x^4 \, dx$$

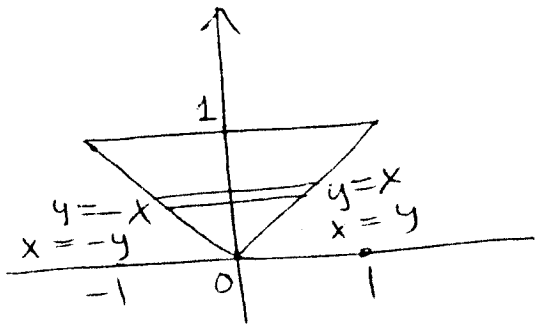
$$= \pi \left[\frac{1}{3}x^3 - \frac{1}{5}x^5 \right]_0^1$$

$$= \pi \left(\frac{1}{3} - \frac{1}{5} \right)$$

$$= \frac{2\pi}{15}$$

4. (20 points.) Find the mass of the triangular region below. All lengths are in meters, and the density of the region is given by $\delta(y) = e^{y^2}$ grams/m^2.





← L(y) →

$L(y) = y-(-y) = 2y$

$dm = \delta(y)\ L(y)\,dy$

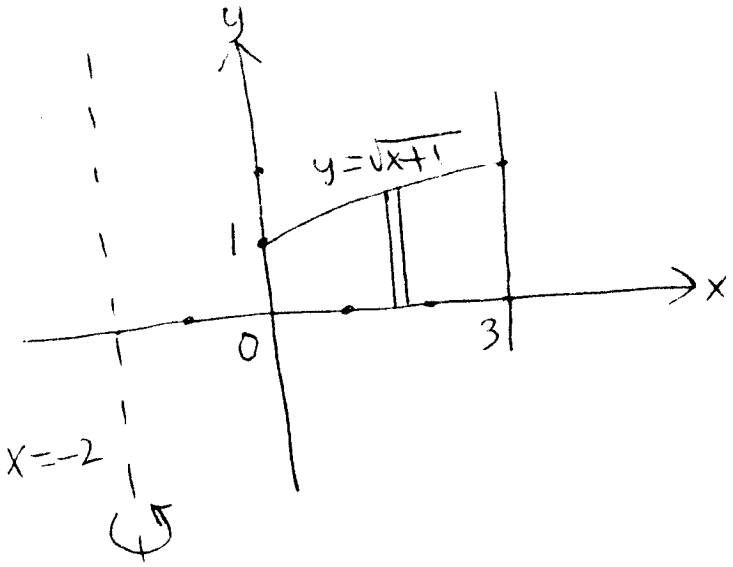
$m = \int dm$

$= \int_0^1 e^{y^2} 2y\,dy$

$= e^{y^2}\Big]_0^1$

$= e^1 - e^0 \quad = e-1$

5. (20 points.) The region bounded by the graphs of $y = \sqrt{x+1}$, $x = 0$, $x = 3$ and the x-axis is revolved about the line $x = -2$. Find the volume of the resulting solid.



$$V = 2\pi \int_0^3 (x+2)\sqrt{x+1}\, dx$$

$$\left[\begin{array}{l} u = x+1, \quad x+2 = u+1 \\ du = dx \end{array}\right.$$

$$= 2\pi \int_1^4 (u+1)\sqrt{u}\, du$$

$$= 2\pi \int_1^4 u^{3/2} + u^{1/2}\, du$$

$$= 2\pi \left[\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right]_1^4 = \frac{512\pi}{15}$$