Math 16A Short Calculus

Final Exam

Name: _____

Signature: _____

Directions: Turn off all cell phones now. Do not open your test until directed. You may not use books, notes, calculators, or cell phones on this test. Read each problem carefully. Make sure you answer every part of each problem. You have 2 hours. *Show your work!*

Good luck!

(scratch page)

1. **Continuity** (60 points)

   For each of the following functions, state: the range, the interval(s) on which the function is continuous, and for each discontinuity whether or not the discontinuity is removable. (60 points, 15 points each)

   (a) $y = x^2 + 4$

   (b) $y = \frac{x+2}{x^2-4}$

   (c) $y = 2\lfloor x \rfloor$

   (d) $y = 4\cos(\theta) + 1$

2. **Slope and Derivative** (50 points)

(a) Find an equation of the line tangent to $f(x) = x^2 + 1$ with slope -2. (20 points)

(b) Use the limit definition to find the derivative of the following functions (30 points; 15 points each)

i. $f(x) = x^2 - 1$

ii. $h(t) = 2\sqrt{t}$

3. **Rates of Change** (45 points)

(a) The profit P from selling x units of a product is given by

$$P = 15 + 12\sqrt{x} - \frac{81}{x}.$$

i. Find the average rate of change of P on the interval $[1, 9]$. (10 points)

ii. Find a formula for the marginal profit. What is the marginal profit for $x = 9$? (15 points)

(b) A sphere of radius 3 has its radius changing at a rate of 4 inches per second. How quickly is the volume of the sphere changing? (20 points)

4. **Derivative Tests** (50 points)

(a) Apply the Second-Derivative Test to find the relative extrema of $f(x) = x^3 - 3x^2 + 1$. Show all of your work. (25 points)

(b) Apply the First-Derivative Test to find the relative extrema of $g(t) = t^4 - 4t^3 - 4$. Show all of your work. (25 points)

5. **Optimization** (50 points)

(a) Find two positive numbers such that their product is 75, and so that the first plus three times the second is a minimum. (20 points)

(b) A rectangular page is to contain 18 square inches of print. The margins at the top and bottom are .5 inches and on each side are 1 inch. What dimensions minimize the amount of paper used? (30 points)

6. **Sketching Graphs** (60 points)

Sketch the graphs of the following functions, showing all work: state domain and range; label all asymptotes, points of discontinuity, and intercepts; and show how you obtained relative extrema, points of inflection. (30 points each)

(a) $g(x) = \frac{x^2}{x^2-1}$

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6. Sketching Graphs (continued)

(b) $f(x) = \frac{x^2-2}{x^2-x-2}$

7. **Differentials** (45 points)

(a) Find the differential dy if $y = \sqrt{x^2 + 1}$. (15 points)

(b) Find the differential dy if $y = \sin\theta + \cos\theta$ (15 points)

(c) Use a differential to approximate the change in revenue R corresponding to an increase in the number of sales x of one unit, if $R = 50x - 2x^2$ and $x = 20$. (15 points)

8. **True/False** (40 points, 5 points each)
Mark each question as (**T**)rue or (**F**)alse.

a. ___ The volume of a sphere of radius r is $4\pi r^3/3$.

b. ___ The absolute maximum of a function on a closed interval never occurs at the endpoints.

c. ___ The derivative of $\sec(x)$ is $-\sec(x)\tan(x)$.

d. ___ A function can only change from increasing to decreasing at a critical number.

e. ___ A function cannot intersect its horizontal asymptotes.

f. ___ If $f'(x) > 0$ for all x, then f is concave upward for all x.

g. ___ The Product Rule states: $\frac{d}{dx}[f(x)g(x)] = g(x)f'(x) + f(x)g'(x)$.

h. ___ A point of inflection of f can occur *only* at a critical number of f'.

Extra credit questions:
State the general form of the equation of a circle. (4 points)

State the definition of the derivative of a function $f(x)$. (4 points)

State the derivative of $|x|$. (4 points)

Problem	Score
# 1	/60
# 2	/50
# 3	/45
# 4	/50
# 5	/50
# 6	/60
# 7	/45
# 8	/40
Total	/400