Math 16A
Final Exam

Printed Name _____

                (FIRST)   (LAST)

Signature   _____

ID Number   _____

Please Show All Your Work, and Mark Your Answers Clearly.
No Calculators – No Scratch Paper – No Cell Phones

Make sure you have **9 pages** of problems. (The last 2 problems are extra-credit.)

You are expected to do your own work, and to adhere to the UCD Code of Academic Conduct.

Simplify all numerical answers.

Give <u>units</u> for your answers where appropriate.

In <u>max-min problems</u>, show whether your answer gives a max. or a min.

In <u>#6bc</u>, use <u>interval notation</u>.

Please indicate clearly if you continue work on the back of a page.

Please stop working **immediately** when time is called.

Have a Good Holiday!

A) $f(x) = \left[\dfrac{8}{x} + (x^2 + 3x + 1)^6 \right]^{10}$

8
PTS

B) $f(x) = \sin^5 \dfrac{x}{3} \tan^7 4x$

9
PTS

C) $f(x) = \dfrac{\sqrt{3x} \, (5x^2 + 2)^8}{(x^4 + 3x + 7)^5}$

9
PTS

① FIND THE FOLLOWING LIMITS, OR SHOW THAT THEY DO NOT EXIST!

A) $\lim\limits_{x \to 9} \dfrac{x^2 - 7x - 18}{\sqrt{x} - 3}$

7
PTS

B) $\lim\limits_{x \to 6} \dfrac{\frac{9}{x^2} - \frac{1}{4}}{x - 6}$

4
PTS

C) $\lim\limits_{x \to -\infty} \dfrac{11x + 5|x|}{\sqrt{4x^2 - x}}$

8
PTS

③ FIND THE SLOPE-INTERCEPT FORM FOR THE TANGENT LINE TO THE GRAPH OF

$$f(x) = \frac{x^2 + 5}{4x - 10} \qquad \text{AT THE POINT } (3, f(3)).$$

8
PTS

④ IF $D_x (\cosh^{-1} x) = \dfrac{1}{\sqrt{x^2 - 1}}$, FIND $h'(x)$ FOR $h(x) = \cosh^{-1}(2 \sec 5x)$

8
PTS

⑤ FIND THE RELATIVE EXTREMA FOR $f(x) = \dfrac{x^2 - 4x + 7}{x - 3}$.

9
PTS

⑥ Let $f(x) = \dfrac{3x^2 - 10x + 3}{(x-1)^2}$, so $f'(x) = \dfrac{4(x+1)}{(x-1)^3}$ and $f''(x) = \dfrac{-8(x+2)}{(x-1)^4}$.

A) FIND EQUATIONS FOR THE ASYMPTOTES TO THE GRAPH OF f.

7 PTS

VERTICAL: _____   HORIZONTAL: _____

B) FIND THE OPEN INTERVALS ON WHICH f IS INCREASING OR DECREASING.

5 PTS

C) FIND THE OPEN INTERVALS ON WHICH THE GRAPH OF f IS CONCAVE UP OR CONCAVE DOWN.

5 PTS

D) SKETCH THE GRAPH OF f, SHOWING ALL ASYMPTOTES, INTERCEPTS, RELATIVE EXTREMA, AND POINTS OF INFLECTION.

10 PTS

(7) FIND $\dfrac{dy}{dx}$ FOR THE EQUATION $x^3 y^5 + 4x^2 = 5y^2 + \cos 3y$.

11
PTS

(8) FIND THE ABSOLUTE EXTREMA FOR $f(x) = 2\sin x - \cos 2x$ ON $\left[0, \dfrac{5\pi}{2}\right]$.

10
PTS

(9) SKETCH THE GRAPH OF A RATIONAL FUNCTION f WITH THE FOLLOWING PROPERTIES:

A) $x = -3$ AND $x = 2$ ARE VERTICAL ASYMPTOTES.

B) $y = x$ IS A SLANTED ASYMPTOTE.

C) $f(3) = 0$ AND $f(-2) = -4$ ARE REL. MIN.

D) $f(1) = 3$ AND $f(-4) = -2$ ARE REL. MAX.

E) $(5,3)$ IS A POINT OF INFLECTION.

9
PTS

(10) IF THE VOLUME OF A CUBE IS INCREASING AT A RATE OF $45 \text{ cm}^3/\text{sec}$, HOW FAST IS ITS SURFACE AREA CHANGING WHEN THE SURFACE AREA IS 150 cm^2?

10
FT3

(11) A CONICAL TANK (WITH VERTEX DOWN) IS 30 FT ACROSS THE TOP AND 90 FT DEEP. IF WATER IS BEING PUMPED INTO THE TANK AT THE RATE OF $20 \text{ FT}^3/\text{MIN}$, FIND THE RATE AT WHICH THE WATER LEVEL IS RISING WHEN THE WATER IS 12 FT DEEP. $\left(V = \frac{1}{3} \pi r^2 h \right.$
$\left. \overline{\text{FOR A CIRCULAR CONE}} \right)$
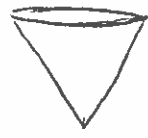
10
FT3

(12) USE THE DEFINITION OF THE DERIVATIVE AS A LIMIT TO FIND $f'(x)$ FOR

$$f(x) = \sqrt{x^3 + 5x^2}.$$

14
PTS

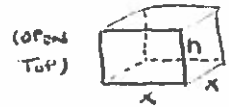(13) FIND THE SLOPE-INTERCEPT FORM FOR THE LINE WHICH IS TANGENT TO THE GRAPH OF

$y = 3\sqrt{x}$ AND WHICH PASSES THROUGH THE POINT $(-4, -8)$.

11
PTS

(14) AN OPEN RECTANGULAR BOX WITH A SQUARE BASE IS TO BE MADE FROM $300 \, in^2$ OF MATERIAL. FIND THE DIMENSIONS THAT WILL GIVE THE LARGEST POSSIBLE VOLUME.

(OPEN TOP)

10 PTS

(15) A COMPANY WANTS TO MAKE A CYLINDRICAL CAN WITH A VOLUME OF $540\pi \, cm^3$. IF THE MATERIAL FOR THE TOP AND BOTTOM COSTS $50¢/cm^2$ AND THE MATERIAL FOR THE SIDE COSTS $40¢/cm^2$, FIND THE RADIUS AND HEIGHT OF THE LEAST EXPENSIVE CAN.

13 PTS

(16) Use a differential to approximate $\sqrt{97}$. (Write your answer as a decimal.)

7
Pts

(17) When a product was sold at \$30 per unit, sales were 400 units per day; but when the price per unit was increased by \$5, sales dropped to 360 units per day. Assuming that the demand function is linear, find the price per unit which will maximize total revenue. (You do NOT have to check that your answer is a maximum.)

9
Pts
(Extra
Credit)

(18) A cyclist is 16 miles north of an east-west road, and his car is $4\frac{1}{2}$ miles east of the point on the road nearest to him. If he bikes 12 mph off the road and 20 mph on the road, find the least amount of time required for him to reach the car.

14
Pts
(Extra
Credit)