1. (30 points) Evaluate each limit.

   (a) $\lim_{x \to \infty} x^2 e^{-x}$

   (b) $\lim_{x \to 0} \dfrac{1 - \cos^2 x}{x}$

(c) $\lim_{x \to -1} \dfrac{\sqrt{x^2+8}-3}{x+1}$

(d) $\lim_{x \to \infty} \dfrac{\cos^2 x}{x}$

(e) $\lim_{x \to 0} \frac{2^x - 1}{5^x - 1}$

(f) $\lim_{x \to 0} (e^x + x)^{1/x}$

2. (20 points) Find $\dfrac{dy}{dx}$. You need not simplify your answer.

(a) $y = (\ln x)^{2x}$

(b) $y = (\ln(1 - x^3))^2$

(c) $y = (3x^2 - 1)(5x^4 - 10x)$

(d) $y = \log_3 \dfrac{1}{x+1}$

3. (15 points) A rectangle has its base on the x-axis, its lower left corner at $(0, 0)$, and its upper right corner on the curve $y = \frac{1}{x}$.

   (a) Express the perimeter of the rectangle as a function of x, namely $p(x)$.

   (b) What is the domain of $p(x)$?

   (c) Find the smallest perimeter of the rectangle.

4. (20 points) Consider the function $f(x) = \dfrac{(x+1)^2}{1+x^2}$

(a) Determine where the function is increasing and where it is decreasing. Find all the critical points if any.

(b) Determine where the function is concave up and where it is concave down. Find all the inflection points if any.

(c) Find all the (vertical, horizontal, oblique) asymptotes if any.

(d) Sketch the graph of $f(x)$. Label all the global and local extrema, inflection points, and the y-intercept.

5. (10 points) Find the equations of the lines that are (a) tangent and (b) normal to the curve

$$6x^2 + 3xy + 2y^2 + 17y - 6 = 0$$

at $(-1, 0)$.

6. (10 points) Assume that the recursively defined sequence $\{a_n\}$ converges:

$$a_0 = 2, \quad a_{n+1} = \frac{72}{1 + a_n}$$

(a) Find a_1, a_2, and a_3. You need not simplify your answer.

(b) Find $\lim_{n\to\infty} a_n$.

7. (6 points) Determine the value of a so that the function $f(x)$ is continuous at $x = 0$.

$$f(x) = \begin{cases} \dfrac{\sin 3x}{5x} & \text{if } x \neq 0 \\ a & \text{if } x = 0. \end{cases}$$

8. (8 points) Show that $x^3 - x - 2 = 0$ has a root in the interval $[0, 2]$. Use the bisection method twice to find the interval of length $\frac{1}{2}$ containing the root.

9. (6 points) If a snowball melts so that its surface area decreases at a rate of 2 cm^2/min, find the rate at which the diameter decreases when the radius of the snowball is 5 cm.