Math 17B
Final Exam

Printed Name _____
(FIRST) (LAST)

Signature _____

ID Number _____

Please Show All Your Work, and Mark Your Answers Clearly.

No Calculators -- No Scratch Paper -- No Cell Phones

There are **8 pages** of problems. (The last problem is for extra credit.)

You are expected to do your own work, and to adhere to the UCD Code of Academic Conduct.

Simplify all numerical answers, except in #13.

In #6 and #9, give units for your answers.

Please indicate clearly if you continue work on the back of a page.

Please stop working **immediately** when time is called.

Have a Good Break!

① a) IF $A = \begin{bmatrix} 5 & 4 \\ 7 & 6 \end{bmatrix}$, FIND A^{-1}.

5 PTS

b) USE YOUR ANSWER TO PART a) TO SOLVE THE LINEAR SYSTEM $5x + 4y = 13$
$7x + 6y = 15$.

3 PTS

② IF $A = \begin{bmatrix} 4 & 7 \\ 1 & -2 \end{bmatrix}$, FIND THE EIGENVALUES AND CORRESPONDING EIGENVECTORS FOR A.

10 PTS

③ IF $L = \begin{bmatrix} 1 & 2 \\ 3/8 & 0 \end{bmatrix}$ IS A LESLIE MATRIX, FIND a) THE LONG-TERM GROWTH RATE AND
b) THE LONG-TERM PERCENTAGE DISTRIBUTION FOR THE 2 AGE CLASSES

8 PTS

④ FIND $\int x^2 \cos 4x \, dx$. (SIMPLIFY THE COEFFICIENTS.)

9 PTS

⑤ A TANK INITIALLY CONTAINS 380 GALLONS OF WATER WITH 16LB OF SUGAR IN SOLUTION. WATER CONTAINING 7 LB OF SUGAR PER GALLON ENTERS THE TANK AT A RATE OF 5 GAL/MIN, AND THE STIRRED MIXTURE IS DRAINED FROM THE TANK AT A RATE OF 8 GAL/MIN. SET UP A DE FOR $A(t)$, THE AMOUNT OF SUGAR IN THE TANK AT TIME t.

8 PTS

⑥ A LAKE IS INITIALLY STOCKED WITH 300 FISH, AND AFTER 5 YRS THERE ARE 400 FISH. IF THE LAKE CAN SUSTAIN A MAXIMUM OF 1200 FISH, AND THE POPULATION INCREASES ACCORDING TO THE LOGISTIC GROWTH MODEL, FIND WHEN THERE WILL BE 800 FISH.

11 PTS

⑦ FIND THE FOLLOWING INTEGRALS:

A) $\int \frac{11x + 19}{(x-1)(x^2+4)}\, dx$

10 PTS

a) $\int x^5 (\ln x)^2\, dx$

9 PTS

⑧ EVALUATE $\int_0^7 \frac{2x}{\sqrt{3x+4}}\, dx$

10 PTS

(9) THE SPEED OF A TURTLE MOVING ALONG A LINE AFTER t HR IS GIVEN BY $f(t) = \dfrac{90t}{(t^2+2)^2}$ FT/HR. FIND ITS AVERAGE SPEED FOR THE FIRST 4 HR.

9 PTS

(10) FIND THE AREA OF THE REGION BOUNDED BY THE GRAPHS OF $y = 2x$, $y = 8$, AND $y = \dfrac{8}{x}$.

9 PTS

(11) FIND THE VOLUME OF THE SOLID GENERATED BY REVOLVING THE REGION BOUNDED BY THE GRAPHS OF $y = 6\sqrt{x}$ AND $y = 3x$ ABOUT THE X-AXIS.

9 PTS

(12) IF $f(x) = \sqrt{x}$, FIND THE 3RD-ORDER TAYLOR POLYNOMIAL $P_3(x)$ ABOUT $a = 100$, AND SIMPLIFY THE COEFFICIENTS.

8 PTS

(13) APPROXIMATE $\int_5^{21} \frac{1}{\ln x}\, dx$ USING $n = 4$ AND (DO NOT SIMPLIFY NUMERICALLY.)

A) THE MIDPOINT RULE.

5 PTS

B) SIMPSON'S RULE.

5 PTS

(14) SOLVE THE DE $y' - 4xy = 24x e^{5x^2}$; $y = 13$ WHEN $x = 0$.

10 PTS

(15) FIND THE ANGLE BETWEEN THE VECTORS $u = \langle 2, 1, 2 \rangle$ AND $v = \langle 4, 8, 1 \rangle$.

7 PTS

(16) USE A ROTATION MATRIX TO FIND THE VECTOR OBTAINED BY ROTATING THE VECTOR $\begin{bmatrix} 10 \\ 8 \end{bmatrix}$ THROUGH AN ANGLE OF $\pi/6$ (COUNTERCLOCKWISE).

7 PTS

(17) EVALUATE $\int_{1/8}^{\infty} \frac{20}{2x^2 + 7x + 3}\, dx$ (OR SHOW THAT IT DIVERGES).

11 PTS

(18) Solve the DE $\frac{dy}{d\tau} = 12\tau y - 6\tau y^2$ for y.

12 pts

(19) Let A be a matrix with eigenvalues $\lambda_1 = 3$, $\lambda_2 = 2$ and $v_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $v_2 = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ as corresponding eigenvectors. Find and simplify $A^4 \begin{bmatrix} -4 \\ 27 \end{bmatrix}$.

9 pts

(20) FIND AN EQUATION OF THE PLANE WHICH PASSES THROUGH $P(4,-1,6)$ AND IS PERPENDICULAR TO THE LINE $X = 8+2T$, $Y = -6+5T$, $Z = 11-3T$.

7 PTS

(21) FIND THE POINT OF INTERSECTION OF THE PLANE $2X - Y + 3Z = 19$ AND THE LINE WHICH PASSES THROUGH $P(3,1,2)$ AND $Q(5,4,1)$.

9 PTS

(22) FIND AN EQUATION OF THE PLANE WHICH CONTAINS THE POINT $P(5,4,2)$ AND THE LINE $X = 1+3T$, $Y = 2+2T$, $Z = 1+4T$. (SIMPLIFY YOUR ANSWER.)

12 PTS (EXTRA CRED.T)