MATH 21A Final Exam

December 7, 2015

Name:_____

ID:_____

Section:_____

DO NOT OPEN THIS EXAM YET

(1) Fill in your name, ID and section number.

(2) This exam is closed-book and closed-notes; no calculators, no phones.

(3) Please write legibly to receive credit. Circle or box your final answers. If your solution to a problem does not fit on the page on which the problem is stated, please indicate on that page where in the exam to find (the rest of) your solution.

(4) You may continue your solutions on additional sheets of paper provided by the proctor. If you do so, please write your name and ID at the top of each of them and staple them to the back of the exam (stapler available); otherwise, these sheets may get lost.

(5) Anything handed in will be graded; incorrect statements will be penalized even if they are in addition to complete and correct solutions. If you do not want something graded, please erase it or cross it out.

(6) Show your work; correct answers only will receive only partial credit (unless noted otherwise).

(7) Be careful to avoid making grievous errors that are subject to heavy penalties.

(8) If you need more blank paper, ask a proctor.

Out of fairness to others, please stop working and close the exam as soon as the time is called. A significant number of points will be taken off your exam score if you continue working after the time is called. You will be given a two-minute warning before the end.

1	2	3	4	5	6	7	8	Total

1. (15 points) Compute the following limits:

a)

$$\lim_{x \to 2} \ln(x^2 + 2x)$$

b)

$$\lim_{x \to 1} \frac{\sin(x) - \sin(1)}{x^2 - 1}.$$

c)

$$\lim_{x \to \infty} \frac{7x - 8x^2 + 5x^3}{2x^3 - x^2 + 6x - 1}$$

2. (15 points) Consider the function $f(x) = e^{x-x^2}$
a) Find the domain and the equations of vertical and horizontal asymptotes
b) Find the first derivative and determine the intervals where the function is increasing/decreasing
c) Find the second derivative and determine the intervals where the function is concave up/down
d) Sketch the graph of this function

3. (15 points) Compute the derivatives of the following functions:

a)

$$f(x) = \sin(x^3 - x)$$

b)

$$f(x) = x \ln(x - 1)$$

c)

$$f(x) = \frac{\sin x}{x + 1}$$

4. (15 points) Find the equation of the tangent line to the graph of the function $f(x) = \ln(2x + 3)$ at $x = -1$.

5. (15 points) Find y' given the equation $x^3 + y^3 + x + y = 1$.

6. (15 points) Find the minimal and maximal values of the function

$$f(x) = x^4 + 3x^2 + 5$$

on the interval $[-2, 2]$.

7. (10 points) Find the maximal area of a rectangle that fits between the graph of the function

$$f(x) = \frac{1}{x^2 + 1}$$

and the x-axis. Assume that one side of the rectangle belongs to the x-axis.

This is a bonus problem. Please start this problem only if you completed the rest of the exam.

8*. (10 points) Find the number of solutions to the equation

$$x^3 + 2x^2 - x - 1 = 0.$$

Justify your answer.