FINAL

March 20, 2019 MAT 21B Winter 2019 Instructor: Sam Fleischer

Name: Signature:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	$\sum$
Your Score																		
Out Of	10	12	12	12	10	12	10	10	8	8	8	10	10	10	40	10	8	200

Trigonometric Identities

Pythagorean Identities	$\cos^2 x + \sin^2 x = 1$ $\quad 1 + \tan^2 x = \sec^2 x$ $\quad 1 + \cot^2 x = \csc^2 x$
Addition Formulas	$\cos(x+y) = \cos x \cos y - \sin x \sin y$ $\quad \sin(x+y) = \sin x \cos y + \cos x \sin y$
Double-Angle Formulas	$\cos(2x) = \cos^2 x - \sin^2 x$ $\quad \sin(2x) = 2 \sin x \cos x$
Power-Reducing (Half-Angle) Formulas	$\cos^2 x = \frac{1}{2}(1 + \cos(2x))$ $\quad \sin^2 x = \frac{1}{2}(1 - \cos(2x))$

Common Summations

$$\sum_{i=1}^{n} k = kn \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

Common Derivatives

$$\frac{\mathrm{d}}{\mathrm{d}x}(x^n) = nx^{n-1} \qquad \frac{\mathrm{d}}{\mathrm{d}x}(e^x) = e^x \qquad \frac{\mathrm{d}}{\mathrm{d}x}(a^x) = a^x \ln a \qquad \frac{\mathrm{d}}{\mathrm{d}x} \ln|x| = \frac{1}{x} \qquad \frac{\mathrm{d}}{\mathrm{d}x}(\sin x) = \cos x \qquad \frac{\mathrm{d}}{\mathrm{d}x}(\cos x) = -\sin x$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(\tan x) = \sec^2 x \qquad \frac{\mathrm{d}}{\mathrm{d}x}(\cot x) = -\csc^2 x \qquad \frac{\mathrm{d}}{\mathrm{d}x}(\sec x) = \sec x \tan x \qquad \frac{\mathrm{d}}{\mathrm{d}x}(\csc x) = -\csc x \cot x$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{\mathrm{d}}{\mathrm{d}x}\left(\tan^{-1} x\right) = \frac{1}{1+x^2} \qquad \frac{\mathrm{d}}{\mathrm{d}x}\left(\sec^{-1} x\right) = \frac{1}{x\sqrt{x^2-1}}$$

Use the backs of the pages for scratch paper. If you need extra scratch paper, raise your hand and ask.
You may not use any notes.
Once you are finished, please check your work and bring the test to me.

"You got this!"

1 [10 points] Use limits of Riemann Sums to calculate $\int_a^b x^2 \mathrm{d}x$.

Hint: $$\int_a^b x^2 \mathrm{d}x = \int_a^0 x^2 \mathrm{d}x + \int_0^b x^2 \mathrm{d}x$$

2 [12 points] Evaluate the following integrals.

(a) [4 points] $$\int_{-\infty}^{0} \pi^{x-1} \mathrm{d}x$$

(b) [4 points] $$\int_{2}^{4} x^{\pi-1} \mathrm{d}x$$

(c) [4 points] $$\int x^2 \left(\frac{x^3}{18} - 1\right)^5 \mathrm{d}x$$

3 [12 points] Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ for the following integral equations.

(a) [4 points] $$y = \int_{0}^{x} \sin(t^2) \mathrm{d}t$$

(b) [4 points] $$y = \int_{\sqrt{x}}^{0} \ln(t^4) \mathrm{d}t$$

(c) [4 points] $$y = \int_0^{e^{x^2}} \frac{1}{\sqrt{t}}\mathrm{d}t$$

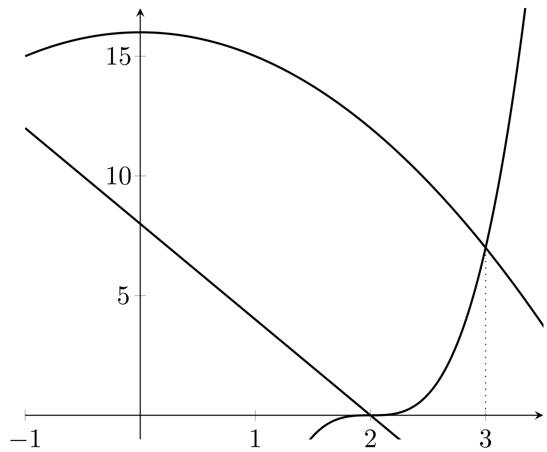
4 [12 points] Evaluate the following integrals.

(a) [4 points] $$\int \csc\left(\frac{x-\pi}{2}\right)\cot\left(\frac{x-\pi}{2}\right)\mathrm{d}x$$

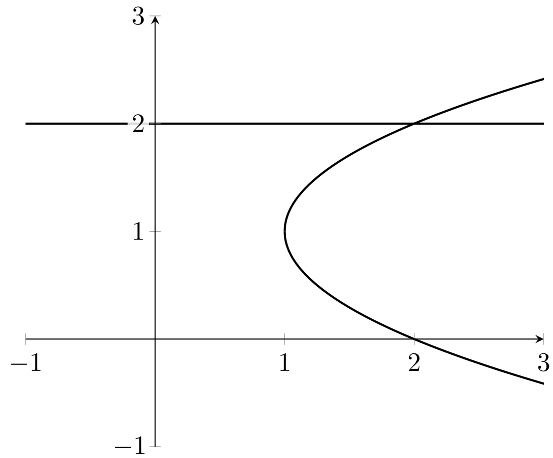
(b) [4 points] $$\int \tan^7\frac{x}{2}\sec^2\frac{x}{2}\mathrm{d}x$$

(c) [4 points] $$\int_{-1}^{1} \sin(3x)\sqrt{e^{x^2}+1}\mathrm{d}x$$

5 **[10 points] Find the area of the region in the first quadrant bounded above by the curve $y = 16 - x^2$, and bounded below by the curves $y = 8 - 4x$ and $y = 7(x-2)^3$**



6 [12 points] Consider the region in the first quadrant bounded by the x- and y-axes, the line $y = 2$, and the parabola $x = y^2 - 2y + 2$. Find the volume of the solid generated by revolving this region about (a) the x-axis and (b) the y-axis.



7 **[10 points] Find the length of the curve given by** $y = \int_0^x \sqrt{\sec^4 t - 1}\,\mathrm{d}t$ **for** $-\dfrac{\pi}{4} \leq x \leq \dfrac{\pi}{4}$**.**

8 **[10 points] Find the area of surface generateed by revolving the curve** $y = \dfrac{x^3}{3}$ **for** $0 \leq x \leq 1$ **about the** x**-axis.**

9 [8 points] Suppose it takes 10 **Newtons to stretch a spring with a natural length of** 7 **meters to a length of** 8 **meters. Calculate the work required to stretch the spring from its natural length to a length of** 9 **meters.**

10 [8 points] A 40**lb weight is attached to the bottom of a rope with length** 50**ft. The rope weighs** 0.5**lb/ft. A worker standing on the top of a** 50**ft building uses the rope to lift the weight from the ground to the top of the building. How much work is done lifting the rope and the weight?**

11 **[8 points] Consider a spherical (not hemispherical) tank with radius 5m. Suppose the tank is filled to a depth of 8m with a liquid weighing 3N/m. Set up, *but do not evaluate*, an integral describing mow much work does is done pumping all of the liquid to a pipe at a height 10m above the top of the tank.**

12 **[10 points] Consider a thin metal rod of length 9 meters lying on its side. The density on the left side of the rod is 2 grams per meter and increases linearly to 3 grams per meter on the right side of the rod. Find the center of mass of the rod.**

13 [10 points] Consider a bacterial population with the property that the rate of growth of the population size is proportional to the population size.

[2 points] What is the differential equation that describes this phenomenon?

[2 points] What is the solution of that differential equation?

[6 points] If it takes three days for the population to double in size, how long does it take the population to triple in size?

14 [10 points] Solve the following separable differential equations. Your final answer should be of the form $y = f(x)$.

(a) [5 points] $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x\sqrt{1 - y^2}$

(b) [5 points] $y^2 \frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2y^3 - 6x^2$

15 [40 points] Evaluate the following integrals.

(a) [5 points] $\int x^2 \sin x \mathrm{d}x.$

(b) [5 points] $\int x^2 \sin x^3 \mathrm{d}x$

(c) [5 points] $$\int \sin^2(2\theta)\cos^3(2\theta)\mathrm{d}\theta$$

(d) [5 points] $$\int_0^\infty xe^{-x}\mathrm{d}x$$

(e) [5 points] $$\int \frac{\sqrt{1-x^2}}{x^4}\mathrm{d}x$$

(f) [5 points] $$\int_0^1 \frac{\theta+1}{\sqrt{\theta^2+2\theta}}\mathrm{d}\theta$$ *hint: this is an improper integral.*

(g) [10 points] $\displaystyle\int \frac{1}{x^4 + x}\mathrm{d}x$ *hint:* $x^3 + 1 = (x+1)(x^2 - x + 1)$.

16 **[10 points] Find an equation for the line tangent to the parametric curve given by $x = t - \sin t$ and $y = 1 - \cos t$ at $t = \dfrac{\pi}{3}$. Then find the value of $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ at that point.**

17 **[8 points] Find the length of the parametric curve $x = \ln(\sec t + \tan t) - \sin t$ and $y = \cos t$ for $0 \leq t \leq \dfrac{\pi}{3}$.**