Math 21B - Fall 2016
Final Exam

Last Name: _____ **First Name:** _____

Section: _____

Directions:

- The use of a cell phone, calculator, laptop or computer is prohibited.
- TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
- Answer all of the questions, and present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer*. You will be graded not merely on the final answer, but on the quality and correctness of the work leading up to it.
- The last page of this exam contains a reference sheet with formulas you may or may not find useful. If you would like, you may disconnect this page from the test.

(1) Write down the general form of the partial fractions expansion of the rational function given below. You do *not* have to solve for the constants!

$$\frac{x^4 + x^3 + x^2 + x + 1}{(x^2+1)^2(x^2-1)^2(x-1)(x-2)^2}$$

(2) Evaluate each of the following integrals.

(a) $\displaystyle\int_{-1}^{1} \frac{1}{x^2}\, dx$

(b) $\displaystyle\int \tan^4(x)\sec^4(x)\, dx$

(c) $\displaystyle\int_1^\infty \frac{\ln(x)}{x^2}\,dx$

(d) $\displaystyle\int_0^2 |x^2-1|\,dx$

(e) $\displaystyle\int x^5\sqrt{1-x^2}\,dx$

(3) Consider the parametric equations

$$\begin{aligned} x(t) &= \cos(t) \\ y(t) &= 2 + \sin(t) \end{aligned}$$

for $0 \leq t \leq \pi$.

(a) Find the length of the curve.

(b) Set up, but *do not evaluate*, an integral representing the area of the surface obtained by revolving the curve about the x-axis.

(c) Where does this curve cross the y-axis? Give your answer in *Cartesian coordinates*.

(4) Set up, but *do not evaluate*, an integral representing the volume of the solid obtained by revolving the region bounded by

$$y = x^2 + 2 \qquad \text{and} \qquad x = (y-2)^2$$

about the line $y = 1$.

(5) Consider the integral

$$\int_0^{\pi} 2\sin(x)\ dx.$$

(a) Compute a left hand sum approximation with $n = 2$ subdivisions.

(b) Compute a midpoint sum approximation with $n = 2$ subdivisions.

(c) Compute a trapezoid sum approximation with $n = 2$ subdivisions.

(6) A machine is pulling 100ft of heavy cable up the side of a building. When the cable has been pulled halfway up, the machine malfunctions. Your instructor happens to be passing by, and offers to single-handedly pull the cable the rest of the way up the building. If the cable weighs 10 lb/ft, how much work is needed for your instructor to pull the cable the rest of the way?

(7) Solve the following initial value problem.

$$\frac{dy}{dx} = 3x^2 e^y \qquad\qquad y(0) = -1$$

(8) Determine which of the following statements are true and which are false. Justify your answers.

(a) If $x(t)$ and $y(t)$ are parametric equations and $\frac{dy}{dx} = t^2 + t$, then $\frac{d^2y}{dx^2} = 2t + 1$.

(b) The center of mass of a thin plate must be on the plate.

(c) For a thin plate whose density function $\delta(x)$ is symmetric about the y-axis, the center of mass must have y-coordinate $\overline{y} = 0$.

(d) $\dfrac{d}{dx}\displaystyle\int_x^2 e^{t^2}\ dt = e^{x^2}$.

(Bonus) Find the volume and surface area of the solid obtained by revolving the region bounded by $y = 1/x$, $x = 1$, and the x axis, about the x-axis. Are your answers surprising?

Trigonometric Identities

$$\begin{aligned}
\sin(A+B) &= \sin(A)\cos(B)+\cos(A)\sin(B)\\
\sin(A-B) &= \sin(A)\cos(B)-\cos(A)\sin(B)\\
\cos(A+B) &= \cos(A)\cos(B)-\sin(A)\sin(B)\\
\cos(A-B) &= \cos(A)\cos(B)+\sin(A)\sin(B)
\end{aligned}$$

$$\begin{aligned}
\sin(2A) &= 2\sin(A)\cos(A)\\
\cos(2A) &= \cos^2(A)-\sin^2(A)
\end{aligned}$$

$$\begin{aligned}
\int \sec(x)\,dx &= \ln|\sec(x)+\tan(x)|+C\\
\int \csc(x)\,dx &= -\ln|\csc(x)+\cot(x)|+C
\end{aligned}$$

Error Estimates

$$|E_T| \leq \frac{M(b-a)^3}{12n^2}, \qquad f''(x)\leq M \text{ for all } x\in[a,b]$$

$$|E_S| \leq \frac{M(b-a)^5}{180n^4}, \qquad f^{(4)}(x)\leq M \text{ for all } x\in[a,b]$$