

21B CALCULUS, SECTIONS A01-A05

Final

Name: _____

ID: _____

Section: _____

1. (5 points) Evaluate the improper Integral

$$\int_0^{\infty} x^2 e^{-3x} dx$$

2. (5 points) Evaluate

$$\int_0^{2\pi} \cos(2x) \sin(x) dx$$

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3. (5 points) Evaluate the indefinite integral

$$\int \frac{2x - 4}{x^2 - x} dx$$

4. (5 points)

$$\int x^3 \sqrt{4 - x^2} dx$$

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5. (5 points) Evaluate the definite integral

$$\int_1^4 \sqrt{t} \ln(t) dt$$

6. (5 points) Evaluate the indefinite integral.

$$\int \frac{\sec^2 \theta \tan \theta}{\sqrt{9 - \tan^2 \theta}} d\theta$$

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7. (10 points) Find the centroid of the triangle bounded by the lines $y = 6x$, $y = 6$, $y = x$.

8. (20 points) Let $f(x)$ be a continuous function which satisfies

$$\int_2^{\ln(y)} f(x) dx = y^3 + C$$

What does C have to be in order to even make this equation solvable? Solve for $f(x)$. Clearly explain your answer.

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9. (20 points) This is a multi-part question. First find the volume of the unbounded solid generated by rotating the curve $y = x^{-2}$ from $x = 1$ to $x = a$, $a > 1$ about the x-axis. Second, find the volume of the solid generated by rotating the same curve about the y-axis. Now take the limit as $a \rightarrow \infty$. Compare the two results and discuss.

10. (20 points) This is a multi-part question. First, derive the trapezoid rule for numerical computation of integrals. Next, consider the integral

$$I = \int_0^{\pi} \sin(x) dx.$$

(i) Find the exact value of the integral. Then, using four subintervals, numerically find the approximate value of the integral using (ii) Rectangles evaluated at the endpoint of the interval, (iii) the trapezoid rule and (iv) Simpson's Rule. Graphically interpret your results. The formula for Simpson's Rule is

$$I \approx \frac{\Delta x}{3} (f_0 + 4f_1 + 2f_2 + 4f_3 + 2f_4 + \dots + 4f_{n-1} + f_n)$$

Compare and discuss your results (you can also use the approximations $\sqrt{2} \approx 1.4$ and $\pi/4 \approx 0.79$).

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1. Please do not turn this exam until instructed to do so.
 2. Write your name, SID, section (as in A01, A02, etc) on each page, and on any scratch paper that you include.
 3. It is a violation of the university honor code to, in any way, assist another person in the completion of this exam. It is a violation of the university honor code to copy answers from someone else's exam. Please keep your own work covered up as much as possible during the exam so that others will not be tempted or distracted. Thank you for your cooperation.
 4. No books, notes, cell phones, classmates may be used for this exam. You may ask us for extra paper.
 5. Read directions carefully. Show all work to receive full credit. A correct answer with no supporting work will NOT receive full credit (so turn in scratch paper if necessary). What you write down and how you write it are the most important means of your getting a good score on this exam. Neatness and organization are also important.
 6. Make sure you have 4 pages (double sided), including this back page (which is numbered page 8). We have scratch paper if you need it.

Problem	points
1 (5 points)	
2 (5 points)	
3 (5 points)	
4 (5 points)	
5 (5 points)	
6 (5 points)	
7 (10 points)	
8 (20 points)	
9 (20 points)	
10 (20 points)	
Total	