Math 22A-2
Final Exam

Printed Name ______________________________
(FIRST) (LAST)

Signature ______________________________

ID Number ________________

Please show all your work, and mark your answers clearly.

No Calculators -- No Scratch Paper

There are **8 pages** of problems. (The last 2 problems are for extra credit.)

You are expected to do your own work, and to adhere to the UCD Code of Academic Conduct.

Simplify all numerical answers.

Please indicate clearly if you continue work on the back of a page.

LI stands for "Linearly independent," and LD stands for "Linearly dependent."

Please stop working **immediately** when time is called.

Have a Good Break!

① LET $A = \begin{bmatrix} 1 & 1 & 3 & 3 & 1 \\ 1 & 2 & 8 & 2 & 8 \\ -1 & 1 & 7 & -4 & 10 \\ 3 & 1 & -1 & 7 & 1 \end{bmatrix}$. GIVEN THAT A HAS RREF $R = \begin{bmatrix} 1 & 0 & -2 & 0 & 6 \\ 0 & 1 & 5 & 0 & 4 \\ 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$,

a) FIND A BASIS FOR THE COLUMN SPACE OF A.

4 PTS

b) FIND A BASIS FOR THE NULLSPACE OF A.

7 PTS

② IF $A = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 3 \\ 1 & 2 & 9 \end{bmatrix}$, FIND A^{-1} USING ELEMENTARY ROW OPERATIONS.

10 PTS

③ IF $A = \begin{bmatrix} 11 & 3 & 5 \\ 7 & 2 & 3 \\ 8 & 4 & 2 \end{bmatrix}$, USE COFACTORS TO FIND THE SECOND ROW OF A^{-1}.

10 PTS

(4) USE THE GRAM - SCHMIDT PROCESS TO CHANGE THE FOLLOWING BASIS FOR $\mathbb{R}^3$ INTO AN ORTHONORMAL BASIS: $u_1 = (1,1,1)$, $u_2 = (6,4,5)$, $u_3 = (7,3,2)$.

10 PTS

(5) IF $L: M_{59} \to P_8$ IS A L.T. WITH RANGE $(L) = P_8$, FIND THE DIMENSION OF $\ker(L)$.

5 PTS

(6) LET $W = \text{SPAN}\{u_1, u_2\}$ IN $\mathbb{R}^4$ WHERE $u_1 = (1,0,2,-2)$ AND $u_2 = (4,3,5,-2)$. IF $u = (11,2,2,3)$, FIND $\text{PROJ}_W u$.

11 PTS

⑦ IF $W = \text{SPAN}\{u_1, u_2\}$ IN $\mathbb{R}^5$ WHERE $u_1 = (1, 2, -4, 3, -3)$ AND $u_2 = (2, 3, -5, 2, -4)$, FIND A BASIS FOR $W^\perp$.

10 PTS

⑧ LET $S = \{v_1, v_2\}$ AND $T = \{w_1, w_2\}$ BE BASES FOR $\mathbb{R}^2$, WHERE $v_1 = (-1, 2)$ AND $v_2 = (3, 5)$. IF $P_{TS} = \begin{bmatrix} 5 & 2 \\ 4 & 2 \end{bmatrix}$ IS THE TRANSITION MATRIX FROM S TO T, FIND THE VECTOR u WITH $[u]_T = \begin{bmatrix} 9 \\ 4 \end{bmatrix}$.

9 PTS

⑨ LET C BE A FIXED 8×8 MATRIX, AND DEFINE $L : M_{58} \to M_{85}$ BY $L(A) = CA^T$. SHOW WHETHER OR NOT L IS A LINEAR TRANSFORMATION.

7 PTS

(10) FIND THE STANDARD MATRIX FOR THE L.T. $L: \mathbb{R}^2 \to \mathbb{R}^2$ WHICH REFLECTS EACH POINT THROUGH THE Y-AXIS, AND THEN ROTATES EACH POINT THROUGH AN ANGLE OF 30° ABOUT THE ORIGIN. ($\sin 30^\circ = \frac{1}{2}$ AND $\cos 30^\circ = \frac{\sqrt{3}}{2}$).

10 PTS

(11) FIND THE LEAST SQUARES SOLUTION OF $Ax = b$, WHERE $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 2 \end{bmatrix}$ AND $b = \begin{bmatrix} 0 \\ 6 \\ 4 \end{bmatrix}$.

9 PTS

(12) FIND A BASIS FOR THE SUBSPACE W OF P_3 GIVEN BY $W = \{ P(T) \text{ IN } P_3 : P(5) = 2P(3) \}$.

10 PTS

(13) LET A BE ANY $m \times n$ MATRIX, AND LET $W = \{x \text{ in } \mathbb{R}^n : Ax = 0\}$.
SHOW THAT W IS A SUBSPACE OF $\mathbb{R}^n$.

7 PTS

(14) FIND ALL VALUES OF C FOR WHICH THE FOLLOWING VECTORS IN $\mathbb{R}^4$ ARE LI
(USING THE DEF.) : $V_1 = (1, 2, 4, -1)$, $V_2 = (1, 3, 6, 4)$, $V_3 = (2, 7, C, 13)$

10 PTS

(15) LET $W = \text{SPAN}\{V_1, V_2, V_3\}$ IN $\mathbb{R}^4$ WHERE $V_1 = (1, 2, 3, -1)$, $V_2 = (2, 5, 8, -5)$, AND $V_3 = (3, 4, 6, 2)$.
FIND ALL VALUES OF d FOR WHICH $u = (5, 1, 4, d)$ IS IN W.

10 PTS

(16) IF $A = \begin{bmatrix} 3 & 2 & 6 \\ 0 & 4 & 3 \\ 0 & -2 & -3 \end{bmatrix}$, FIND AN INVERTIBLE MATRIX P AND A DIAGONAL MATRIX D WITH $P^{-1}AP = D$.

14 PTS

(17) IF A IS A 2×2 MATRIX SUCH THAT $Av = 2v$ AND $Aw = -w$ WHERE $v = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ AND $w = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, CALCULATE A^6.

11 PTS

(18) IN EACH OF THE FOLLOWING CASES, FIND THE RANK OF A (OR WRITE "I" IF THE STATED CONDITION IS IMPOSSIBLE.)

5 PTS

_____ a) A IS AN 8×10 MATRIX WITH LI ROWS.

_____ b) A IS A 5×8 MATRIX WITH LI COLUMNS.

_____ c) A IS A 9×11 MATRIX WITH A 7-DIM. NULLSPACE.

_____ d) A IS A 15×10 MATRIX SUCH THAT $Ax = 0$ HAS ONLY THE TRIVIAL SOLUTION.

_____ e) A IS A 7×9 MATRIX SUCH THAT $Ax = b$ IS CONSISTENT FOR EVERY b IN $\mathbb{R}^7$.

(19) LET $S = \{v_1, v_2\}$ BE THE BASIS FOR $\mathbb{R}^2$ GIVEN BY $v_1 = (4,9)$ AND $v_2 = (2,5)$. IF $L: \mathbb{R}^2 \to \mathbb{R}^2$ IS THE L.T. WITH $[L]_S = \begin{bmatrix} 3 & 6 \\ 5 & -2 \end{bmatrix}$, FIND A FORMULA FOR $L(x,y)$.

14 PTS

(20) FIND A BASIS FOR THE SUBSPACE W OF M_{22} CONSISTING OF ALL 2×2 MATRICES WHICH HAVE $X = \begin{bmatrix} 1 \\ 8 \end{bmatrix}$ AS AN EIGENVECTOR.

10 PTS

(21) LET S BE A SET OF VECTORS IN A VECTOR SPACE V.
IN EACH OF THE FOLLOWING CASES, INDICATE WHETHER S IS
<u>LI</u>, <u>LD</u>, OR COULD BE EITHER (<u>E</u>) BY PLACING THE BEST ANSWER IN THE BLANK.

7 PTS

______ a) S CONTAINS THE ZERO VECTOR.

______ b) S IS A SET OF 5 VECTORS IN $\mathbb{R}^8$.

______ c) S IS A SET OF 7 VECTORS IN $\mathbb{R}^5$.

______ d) S IS A SET OF 2 VECTORS, NEITHER OF WHICH IS A MULTIPLE OF THE OTHER.

______ e) S IS A SET OF 5 VECTORS IN $\mathbb{R}^5$ WHICH DOES <u>NOT</u> SPAN $\mathbb{R}^5$.

______ f) S IS A SET OF 8 VECTORS, NONE OF WHICH IS A LINEAR COMBINATION OF THE OTHERS.

______ g) $S = \{v_1, v_2, v_3\}$ IN $\mathbb{R}^3$ AND $\det(A) \neq 0$ WHERE $A = [v_1 | v_2 | v_3]$.

(22) FIND THE STANDARD MATRIX FOR THE L.T. $L: \mathbb{R}^2 \to \mathbb{R}^2$ WHICH REFLECTS EACH POINT THROUGH THE LINE $Y = \frac{4}{3}X$.

10 PTS

(EXTRA CREDIT)

(23) LET V BE A VECTOR SPACE WITH BASIS $S = \{v_1, \ldots, v_n\}$, AND DEFINE A L.T. $L: \mathbb{R}^n \to V$ BY $L(c_1, \ldots, c_n) = c_1 v_1 + \cdots + c_n v_n$.

a) PROVE THAT L IS 1-1.

8 PTS

(EXTRA CREDIT)

b) PROVE THAT L IS ONTO.