Name : **Student id :**

MAT 22A : Linear algebra

December 13, 2013

1. (a) Compute the projection of the vector $\begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \end{pmatrix}$ to the linear subspace $X = \{x \in \mathbb{R}^4 \mid x_1 = x_2 \text{ and } x_3 + x_4 = 0\}$.

   (b) Over a flat and long drive, a driver manages to link his car's gas mileage to his average speed. In particular he collects 4 data given as follows

Trip Number	1	2	3	4
Average speed (mph)	60	64	62	60
Gas mileage (mpg)	50	47	49	51

   Find the best linear relation that links the gas mileage to the average speed.
   Hint : To simplify the computation, try to fit the relation $(mpg - 50) = C + D(mph - 60)$.

Name : **Student id :**

1.

Name : **Student id :**

MAT 22A : Linear algebra

December 13, 2013

2. Consider the matrix $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \\ 2 & 2 & 0 \end{pmatrix}$.

   (a) Prove that the columns of A are linearly independent.

   (b) Compute an orthogonal basis of $C(A)$, the column space of A

   (c) Compute a vector that is in the orthogonal complement of $C(A)$.

Name : **Student id :**

2.

Name : **Student id :**

MAT 22A : Linear algebra

December 13, 2013

3. Let $A = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & -\frac{1}{2} \\ 1 & -\frac{1}{2} & 1 \\ \frac{1}{2} & -\frac{3}{4} & \frac{3}{2} \end{pmatrix}$.
   (a) Prove that the rank of A is 2.
   (b) Prove that $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ is an eigenvector of A.
   (c) Compute all eigenvalues and eigenvectors of A.
   (d) Compute A^{200} with 5 significant digits.

Name : **Student id :**

3.

Name : **Student id :**

MAT 22A : Linear algebra

December 13, 2013

4. Give a short answer and a justification to the following questions. The justifcation is more important than a correct answer.

   (a) What is the determinant of $\begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 2 \end{pmatrix}$?

   (b) True or false ? A matrix is diagonalizable if and only if, for every eigenvalue, the geometric multiplicity is less than or equal to the algebraic multiplicity.

   (c) Given n pairs $(a_i, b_i), i = 1, \dots, n$, explain how to fit a relation of the type $b \approx Ca^D$ using linear least-squares.

   (d) True or false ? If A has eigenvalues $1, 2, 3$ then A^{-1} has eigenvalues $1, \frac{1}{2}, \frac{1}{3}$.

   (e) True or false ? $\{(1\ 0\ 0), (0\ 1\ 0), (0\ 0\ 1)\}$ is an orthonormal basis for the row space of $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 3 \\ 0 & -1 & 1 \end{pmatrix}$.

   (f) True or false ? If Q is an orthogonal matrix, then Q^T is an orthogonal matrix too.