Math 127-A **Name:** ____________________
Winter 2020
Final
3/17/20
Time Limit: 2 Hours

NOTE: This exam was given remotely during the Winter Quarter of 2020. Students were allowed to use their notes, submitted homeworks, and textbook, but nothing else.

You must answer Problem 1 and Problem 12. For Problems 2-11, you must answer 7 of them. Thus, the total number of points for the exam is 80.

NOTE: Students were given 10-15 minutes extra before the official exam start time to do Problem 1.

1. (5 points) Copy the following statement and write your signature underneath: "As a student at UC Davis, I hold myself to a high standard of integrity and by signing/accepting the statement below I reaffirm my pledge to act ethically by honoring the UC Davis Code of Academic Conduct. I will also encourage other students to avoid academic misconduct.

   I acknowledge that the work I submit is my individual effort. I did not consult with or receive any help from any person or other unauthorized source. I also did not provide help to others.

   I understand that suspected misconduct on this assignment/exam will be reported to the Office of Student Support and Judicial Affairs and, if established, will result in disciplinary sanctions up through Dismissal from the University and a grade penalty up to a grade of 'F' for the course.

   I understand that if I fail to acknowledge or sign this statement, an instructor may not grade this work and may assign a grade of '0' of 'F'."

2. (10 points) Suppose $f : A \to \mathbb{R}$ is a function and c is a limit point of A. Show that if $\lim_{x\to c} f(x) = L_1$ and $\lim_{x\to c} f(x) = L_2$, then $L_1 = L_2$. (In other words, if a functional limit exists, then it is unique.)

3. (10 points) Is the converse of the Monotone Convergence Theorem true? If so, prove it. If not, then provide a counterexample and explain why it is a counterexample.

4. (10 points) Use the Order Limit Theorem to show that $[1, \infty)$ is closed.

5. (10 points) Show that $\lim_{x\to 0} 1/x$ does not exist. Use the definition and/or tools developed in this class.

6. (10 points) Use the sequential characterization of continuity to show that if $f : A \to \mathbb{R}$ and $g : A \to \mathbb{R}$ are continuous at $c \in A$, then $f(x)g(x)$ is continuous at c.

7. (10 points) (a) (5 points) Give an example of a set F which is closed, but not compact, offering a brief justification for why it is closed and a brief justification for why it is not compact.

   (b) (5 points) Give an example of an open cover of F which has no finite subcover.

8. (10 points) Prove, using the definition of a closed set, that if A and B are closed, then $A \cup B$ is closed. You may NOT use the result that the complement of a closed set is open. However, you may use whichever characterization of limit points you desire.

9. (10 points) Prove that if A and B are connected subsets of $\mathbb{R}$, then $A \cap B$ is connected. Hint: There are multiple characterizations of connected sets you may use.

10. (10 points) Is $1 - x^2$ uniformly continuous on $(2, 4)$? Prove or disprove.

11. (10 points) Does $\sum_{n=1}^{\infty} \frac{1}{n!}$ converge or diverge? Justify your answer. (Recall that $n! = 1 \cdot 2 \cdot 3 \cdot ... \cdot n$ for $n \in \mathbb{N}$.)

12. (5 points) Pick a problem on this exam that you found challenging, yet still submitted an answer to. Describe why you found it challenging and how you responded to the difficulty. Your answer should be 4-6 sentences.