Name: ____________________

Final

Wait! Do not turn this page until told to.

No books, notes, phones, or calculators.

Show all of your work.

Justify every statement that you make.

Good luck!

1	2	3	4	5	

for office use

1. Two days after a person is infected with the flu virus, he/she transmits it to six others every day.
   Starting with one person, infected on day zero, let f_n be the number of people having flu on day n. Thus $f_0 = 1$, $f_1 = 1$, $f_2 = 7$.
   (a) Write a recurrence formula for this sequence, and find f_4.

   (b) Find f_n in terms of n.

2. (a) Prove that if 81 numbers are chosen from the set $\{1, \ldots, 100\}$, then there are always five consecutive numbers.

(b) Show that this claim is not true if 80 numbers are chosen.

3. You are given a white fence with $8n$ posts, and four cans of paint: red, green, blue and yellow.

   (a) In how many ways can you paint the fence, such that $2n$ posts are red, $2n$ are green, $2n$ are blue, and $2n$ are yellow?

   (b) In how many ways can you paint n posts with each color, and leave $4n$ posts unpainted?

   (c) Use Stirling's formula to compare the two cases as $n \to \infty$.

4. Consider the complete graph K_5 with labeled vertices $\{1, 2, 3, 4, 5\}$.

In this question you don't have to justify your answers.

(a) Is K_5 connected? ______________

(b) Is K_5 Eulerian? ______________

(c) How many Hamilton cycles does K_5 have? ______________

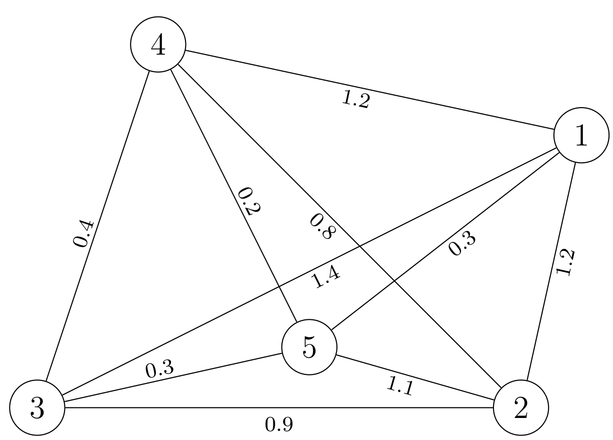
(d) How many perfect matchings does K_5 have? ______________

(e) How many spanning trees does K_5 have? ______________

(f) Find a minimum spanning tree in the following weighted K_5.

What's its total weight? ______________

Write its Prüfer code: ______________



5. (a) Let T be a tree. Prove the following claims.

1. Adding an edge $e \notin T$, to T, creates a cycle $C \subseteq T \cup \{e\}$.

2. Removing an edge $e' \in C$, from $T \cup \{e\}$, yields a tree $T' = T \cup \{e\} \setminus \{e'\}$.

(b) Let T be a minimum spanning tree in a weighted graph (V, E, w).

Suppose that $e \notin T$ and let e' be an edge on the unique path in T that connects the endpoints of e. Show that $w(e') \leq w(e)$.