MATH 150A Final Exam
December 9, 2015

Name:____

ID:____

Section:______________________________________

DO NOT OPEN THIS EXAM YET

(1) Fill in your name, ID and section number (A01 or A02).

(2) This exam is closed-book and closed-notes; no calculators, no phones.

(3) Please write legibly to receive credit. Circle or box your final answers. If your solution to a problem does not fit on the page on which the problem is stated, please indicate on that page where in the exam to find (the rest of) your solution.

(4) You may continue your solutions on additional sheets of paper provided by the proctor. If you do so, please write your name and ID at the top of each of them and staple them to the back of the exam (stapler available); otherwise, these sheets may get lost.

(5) Anything handed in will be graded; incorrect statements will be penalized even if they are in addition to complete and correct solutions. If you do not want something graded, please erase it or cross it out.

(6) Show your work; correct answers only will receive only partial credit (unless noted otherwise).

(7) Be careful to avoid making grievous errors that are subject to heavy penalties.

(8) If you need more blank paper, ask a proctor.

Out of fairness to others, please stop working and close the exam as soon as the time is called. A significant number of points will be taken off your exam score if you continue working after the time is called. You will be given a two-minute warning before the end.

1	2	3	4	5	6	7	8	9	Total

1. (15 points) Consider the permutation

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 7 & 9 & 5 & 2 & 1 & 10 & 3 & 8 & 4 & 6 \end{pmatrix}$$

a) Decompose f into non-intersecting cycles
b) Find the order of f
c) Find the sign of f
d) Find f^{-1}

2. (15 points) Are the following matrices orthogonal? Do they preserve the orientation? Please explain your answers.

$$\text{(a)} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad \text{(b)} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

3.(10 points) Let M_k be the set of all 2×2 matrices with determinant k. For which k M_k is a group?

4. (15 points) Solve the equations

(a) $2x = 7 \mod 11$

(b) $2x = 7 \mod 12$

5. (10 points) f is a homomorphism from a group G_1 with 15 elements to a group G_2 with 13 elements. Prove that f is trivial.

6. (10 points) Compute $2^{100} \mod 5$.

7.(15 points) The base of a triangular prism is a regular triangle with side 1, the height equals 2. Let G be the group of isometries of this prism, consider the action of G on the set of faces.

a) Find the orbits of this action.

b) Find the stabilizers for this action.

c) Use the counting formula to compute the number of elements in G.

8. (10 points) Find all homomorphisms from $\mathbb{Z}_6$ to itself. For each of them find the kernel and the image. How many automorphisms (that is, isomorphisms to itself) does $\mathbb{Z}_6$ have?

This is a bonus problem. Please start this problem only if you completed the rest of the exam.

9*. (10 points) Construct a surjective homomorphism from S_4 to S_3 and describe its kernel.