University of California, Davis
Department of Mathematics

Winter Quarter 2021
MAT 180
Special Topics

Course Title: Linear and Nonlinear Partial Differential Equations of Geophysical Fluid Dynamics
Professor: Joseph Biello

Prerequisites:
- MAT 118A

Text:
Readings will be drawn from the following:
- A.J. Majda "Introduction to PDEs for the Atmosphere and Ocean"
- Vallis "Atmosphere and Ocean Fluid Dynamics
- Research articles by Majda, Biello, Hunter

Course Description:
The intellectual objective of this class is to explore nonlinear waves in rotating fluid dynamics. The pedagogical objective of this class is to lead students into a new field of research and begin to demonstrate what it takes to create a research article. This class will satisfy the capstone requirement and is suited for applied math majors and physics majors, as well as other natural scientists who meet the prerequisites. MAT 118A is a prerequisite as some knowledge of the Fourier series and the wave equation is essential.

We will discuss:
- The linear wave equation (as a review)
- The behavior of waves in environments which vary (such as the continental shelf of the ocean, or the mantle and crust of the earth)
- Linear systems of PDE which describe wave phenomena (especially tropical atmospheric waves)

Other topics discussed include:

- Energy conservation
- Dispersion relations
- WKB approximation
- Asymptotic analysis of weakly non-linear waves

The guiding themes for this course will be

- Geostrophic balance
- Tropical wave dynamics
- The simplified equation systems resulting from these theories

Students will explore examples in detail regarding non-linear waves through:

- The Korteweg de-Vries equation
- The Biello-Majda equations
- The Hunter-Biello equations
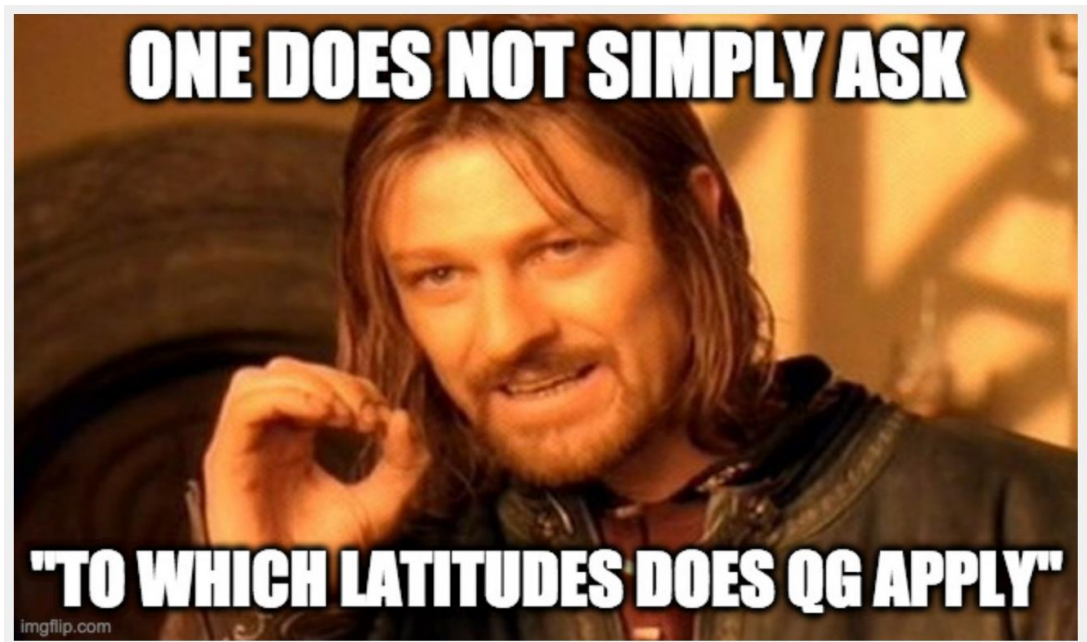
Course Grade:

The grading criteria for this course is still to be determined. However, students will be expected to produce LaTeXed course notes with comments and solved problems every 2 weeks. There will be a final group project structured like a journal article. Though it will not be required to produce new results for the project, some projects will have the opportunity to do so. The final project will need both, mathematical analysis, some numerics, as well as clear and concise writing and relevant citations.