Craig A. Tracy May 12, 2018

Math 180, Spring 2019
Introduction to Analytic Number Theory

Location & Time: TBA, 3:10–4:00 PM, MWF.

> Prime numbers are the multiplicative building blocks of natural numbers. Understanding their overall influence and especially their distribution gives rise to central questions in mathematics and physics. In particular their finer distribution is closely connected with the Riemann hypothesis, the most important unsolved problem in the mathematical world.
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> Hugh L. Montgomery and Robert C. Vaughan
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> *Multiplicative Number Theory I. Classical Theory*

Prerequisites: MAT 185A (Complex Variables) or equivalent course.

Course Grade: 40% homework and 60% final exam.

Textbook: There is no required textbook for the course. Lecture notes will be distributed. A recommended reading list is posted on UC Davis Canvas. These recommended books are on reserve in Shields Library.

Course Outline:

1. Review of complex analysis: Basic results, Fourier transform, Mellin transform. Review of Abel partial summation and the Euler-Maclaurin summation formula.
2. Dirichlet series and arithmetical functions.
   (a) Riemann zeta-function and Euler product formula.
   (b) Analytic continuation of the Riemann zeta-function.
   (c) Functional equation for the Riemann zeta-function.
   (d) Zero-free theorem and zeta-function estimates.
   (e) Perron's formula.
3. Prime number theorem.
4. Partitions of integers
   (a) Dedekind eta function
   (b) Kronecker limit formula
5. Elliptic and modular functions
   (a) Field of elliptic functions
   (b) Weierstrass elliptic function $\wp(z)$.
   (c) Eisenstein series
   (d) Modular group, modular forms, and cusp forms. Dimension formulas
   (e) Number theoretic consequences