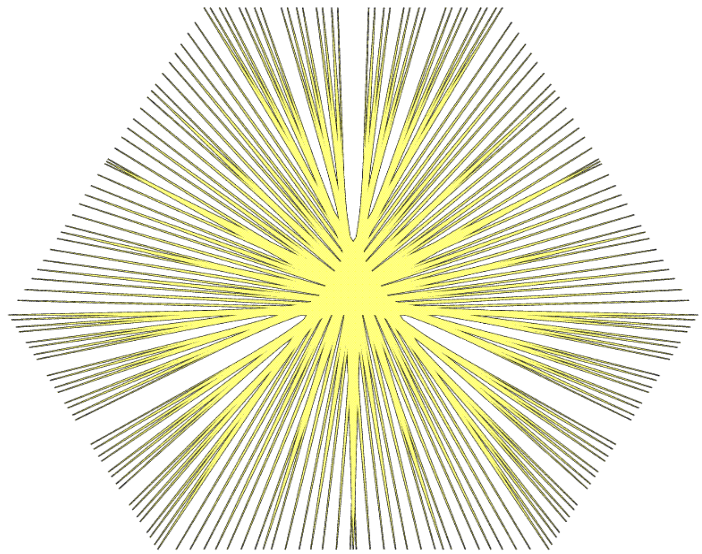
Math 180 Power of Analysis, Winter 2016, Instructor: Kevin Luli

In this course, I want to survey some novel ideas in mathematical analysis (a.k.a calculus). These ideas in turn solve some very interesting elementary problems. Students should have a basic understanding of advanced calculus (at the level of Math 25A) and some exposure to complex analysis. Students will be required to give a presentation on a problem they find interesting and that fits into the theme of the course.



List of selected topics:

- **Kakeya needle problem:** What is the smallest region (in terms of area) in the plane in which one can rotate a needle of unit length by 360 degrees? It turns out you can make the set as small as you want! Understanding the structures of these sets in higher dimension is an active research area that attracts some of the best mathematicians of our time.

- **How do we know pi is irrational?** There are certain people that can recite the first one thousand digits of pi but see no repeated patterns and declare the number to be irrational. But we will see precisely by using tools from analysis why pi cannot be of the form a/b where a and b are integers.

- **Sum of inverse powers of integer?** In Math 21C, we know that the infinite series 1/n^k (sum over k) converges when k>1. The question that people are curious about but the instructors are afraid to address is what they converge to. For even numbers k, we can compute them precisely; it turns out they are some powers of pi, e.g, when k = 2, the sum is pi^2/6. For odd numbers, the problem beats the best minds of our time.

- **Waring's Problem:** Lagrange showed in 1770 that every positive integer is the sum of four perfect squares. Waring conjectured that every positive integer of the sum of a finite number of powers of natural numbers. He conjectured very precisely (albeit with no reason) the number of powers, for example, 9 cubes.

- **the hairy ball theorem:** you can't comb a hairy ball flat without creating a cowlick! Minor gave a very elegant proof based on vector calculus.

- **Integer rectangles:** Suppose we can tile up a rectangle with rectangles all of which have at least one side of integer length. Then the rectangle has at one side of integer length. The proof of this result elegantly makes use of calculus.

- **Polya's polynomial projection theorem:** Consider a polynomial P(z) over the complex plane. Let C be set of all z such that |P(z)| < 2. What can we say about C? Polya gave a very interesting description: In every direction you project onto a line, the shadow of C on the line has length at most 4. The proof makes use of another result of independent interest: Chebyshev's Theorem for polynomial which gives a lower bound on the maximum of a polynomial over the unit interval in terms of the degree.

- **Banach-Tarski paradox:** Start with a watermelon. We can cut the watermelon up into pieces and re-assemble the pieces back together to get two watermelons, each of which has the same volume as the one we started! (No Typos! No Kidding! But it's a mathematical watermelon!)

Other topics of equal entertaining value will be explored as time permits.

Textbooks: "Proof from the book," by Martin Aigner. "A View from the Top: Analysis, Combinatorics and Number Theory" by Alex Iosevich. The American Mathematical Monthly available online.