MAT 280 Harmonic Analysis on Graphs & Networks Syllabus Page (Fall 2019)

Course: MAT 280: Title: Harmonic Analysis on Graphs & Networks
CRN: 48880
Credit Units: 3
Class: TR 1:40pm-3:00pm, 2112 Math. Sci. Bldg.

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Office Hours: By appointment

Synopsis:

Graphs and networks have been successfully used in a variety of fields (e.g., machine learning, data mining, image analysis, sensor networks, social sciences, etc.) that are confronted with the analysis and modeling of high-dimensional datasets. Harmonic analysis tools originally developed for Euclidean spaces and regular lattices are now being transferred to the general settings of graphs and networks in order to analyze geometric and topological structures, and data and signals measured on them. In this course, we shall discuss a variety of important theories and interesting applications employing harmonic analysis *of* and *on* graphs and networks. Topics include: graph Laplacians, their eigenvalues and eigenvectors for structural/morphological analysis; wavelets on graphs; random walks and diffusion on graphs; spectral clustering; community detection; etc.

Prerequisite:

MAT 129, 167, 271, or consent of the instructor.

Topics: I plan to cover the following (subject to change):

- Overture: motivations, scope and structure of the course
- Prelude to Analysis on Graphs: Laplacian Eigenfunctions on General Shape Domains in $\mathbf{R}^d$
- Basics of Graph Theory: Graph Laplacians
- How to Construct Graphs from Given Datasets?
- Distances and Weights of Graphs
- Spectral Clustering of Massive Data
  - Review on PCA & MDS
  - Laplacian Eigenmaps & Diffusion Maps
- Graph Partitioning
- Community Detection
- Fast Algorithms on Graphs
- Wavelets on Graphs
- Graph Embeddings

Textbooks/References:

No textbook is required. Many journal papers will be discussed in the class and their links will be posted in the comments, handouts, and reference page. Yet, the following books and papers may be useful as general introductory references in this field.

- For Laplacians on graphs:
  - D. Cvetković, P. Rowlinson, and S. Simić: *An Introduction to the Theory of Graph Spectra*, Vol. 75, London Mathematical Society Student Texts, Cambridge Univ. Press, 2010.
  - F. R. K. Chung: *Spectral Graph Theory*, AMS, 1997.

  - A. E. Brouwer and W. H. Haemers: *Spectra of Graphs*, Springer, 2012.
  - R. B. Bapat: *Graphs and Matrices*, 2nd Ed., Universitext, Springer, 2014.
  - D. Spielman: "Spectral graph theory," in *Combinatorial Scientific Computing* (O. Schenk, ed.), Chap.18, pp.495-524, CRC Press, 2012.
- For Laplacians on Euclidean domains:
  - W. A. Strauss: *Partial Differential Equations: An Introduction*, 2nd Ed., Chap. 10 & 11, John Wiley & Sons, 2008.
  - R. Courant and D. Hilbert: *Methods of Mathematical Physics, Vol. I*, Chap. V, VI, & VII, First English Edition, John Wiley and Sons, 1953. Republished as Wiley Classics Library in 1989.
- For Laplacians on Riemannian manifolds:
  - S. Rosenberg: *The Laplacian on a Riemannian Manifold*, Vol. 31, London Mathematical Society Student Texts, Cambridge Univ. Press, 1997.
- For Graph Signal Processing:
  - D. I. Shuman, S. K. Narang, P. Frossard, A. Ortega, and P. Vandergheynst: "The emerging field of signal processing on graphs: Extending high-dimensional data analysis to networks and other irregular domains," *IEEE Signal Processing Magazine*, vol.30, no.3, pp.83-98, 2013.
- For Spectral Clustering:
  - U. von Luxburg: "A tutorial on spectral clustering," *Statistics and Computing*, vol. 17, no. 4, pp.395-416, 2007.

Class Web Page:

I will maintain the Web pages for this course (one of which you are looking at now). In particular, please read the comments, handouts, and reference page often. After each class, I will put relevant comments and references as well as most of my handouts in class in this page that should serve as a guide to further understanding of the class material.

Grading Scheme:

- 100% Final Report (due 5pm, Thursday, Dec. 12, 2019)

Final Report:

Your grade will be determined by your final report. My suggestion for writing your report is the following:

- Describe how some of the methods you learned in this course will be used in your research; or
- Find out a practical application dealing with data analysis on a graph or network yourself (not copying from papers/books) using the methods you learned in this course; describe how to use them; describe the importance of that application; what impact would you expect if you are successful?

I will be available for further individual discussions for you to determine the contents of your final report.

Please email me if you have any comments or questions!
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