MAT 280 Winter 2016 ``Geometric & Topological Combinatorics with a view toward applications"

DESCRIPTION: This course will be an introduction to (some) techniques from combinatorial geometry and topology that have proved useful
to solve combinatorial problems arising in graph theory, game theory, mathematical economics, optimization, and statistics.
Examples include the use of Borsuk-Ulam theorem to prove the Kneser graph coloring conjecture, Lov\'asz
Evasiveness of graph properties, or the powerful use of Fixed point theorems to solve problems about dividing fairly a cake or
designing voting schemes. My plan is to cover the following 3 topics. I will develop more or less from scratch the necessary background.

SYLLABUS: Topics will be covered in roughly the following order:
-Part I (weeks 1-3): Combinatorial topology lemmas of Sperner and Fan-Tucker and their dramatic consequences: Polytopal
Sperner theorems (three versions), Brouwer Fixed point theorems, Kakutani's Theorem, Borsuk-Ulam, Van Kampen-Flores,
Lusternik-Schnirelmann-Borsuk, Markov-Kakutani fixed point theorems.

-Part II (weeks 4-7): Combinatorial Geometry tools: Helly-type results, Carath\'eodory-style theorems, Radon-Tverberg theorems.
Weyl-Minkowski's theorem. Separation (e.g, Hahn-Banach, Farkas, KKT), Covering (KKM-Lebesgue covering theorems and their meaning).

-Part III (weeks 8-10) Applications to
Game Theory-Economics (Minmax theorems, Nash-Equilibria, Fair-Division problems, voting theory, HEX),
Data point analysis (Tucker's depth, center points, Ham-sandwich theorem, e-nets),
Optimization (optimality conditions and duality, discrete optimization, chance-constrained optimization, etc),
Combinatorics/Hypergraphs (submodular functions, coloring problems, cores, embeddability of graphs).

GRADING: I believe that ``no pain, no gain" is fairly accurate when it comes to learning mathematics. Thus, I will assign regular problems sets from which
there will be 3 take-home exams, one at the end of each portion above. Also, I will not use a single book (it does not exist), thus students will be asked to serve
as scribes and provide very careful edited notes in latex to everyone else as part of their grade. This course should be suitable for grad students with solid
mathematical background and maturity.

TIME: The time day for the course will be decided in agreement with those students who are planning to take the course for credit, but I am partial
to Tu/Th afternoons .