ALGEBRA REVIEW
LEARNING SKILLS CENTER

The "Review Series in Algebra" is taught at the beginning of each quarter by the staff of the Learning Skills Center at UC Davis. This workshop is intended to be an overview of the algebra skills needed in most college level classes. It was designed for the student who has had algebra before and needs a reminder of the basic principles and rules.

Exponents & Radicals

Exponents are simply a way to abbreviate writing out a long multiplication problem. In the exponential expression b^n , b is called the *base* and n is called the *exponent* or *power.* The expression b^n is called the n^{th} power of b.

Properties of Exponents:

If b is a real number and if n is a positive integer, then

1. Whole number exponents: $b^n = b \cdot b \cdot b \cdot b \cdots b$ (n times)
2. Zero exponent: $b^0 = 1;\ b \neq 0$
3. Negative exponents: $b^{-n} = \dfrac{1}{b^n}$; $b \neq 0$
4. Rational exponents (n^{th} root): $\sqrt[n]{b} = b^{\frac{1}{n}}$; $n \neq 0$, and if n is even, then $b \geq 0$.
5. Rational exponents: $\sqrt[n]{b^m} = \left[\sqrt[n]{b}\right]^m = \left[b^{\frac{1}{n}}\right]^m = b^{\frac{m}{n}}$
   $n \neq 0$, and if n is even, then $b \geq 0$

Operations with Exponents:

1. Multiplying like bases: $b^n \cdot b^m = b^{n+m}$ *Add exponents*
2. Dividing like bases: $\dfrac{b^n}{b^m} = b^{n-m}$ *Subtract exponents*
3. Exponent of exponent: $\left(b^n\right)^m = b^{n \cdot m}$ *Multipy exponents*
4. Removing parentheses: $(ab)^n = a^n \cdot b^n$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

	Property	Watch out!
5. Special conventions:	$-b^n = -(b^n)$	$-b^n \neq (-b)^n$
	$k\,b^n = k(b^n)$	$k\,b^n \neq (kb)^n$
	$(b)^{n^m} = b^{(n^m)}$	$b^{n^m} \neq (b^n)^m$

Exponents and Radicals: Practice Exercises

Simplify the following as much as possible:

1. $3^2 \cdot 3^4 \cdot 3^{-5}$

2. $(-2x^2 y^3)^3$

3. $\left[\frac{7^0}{7^2}\right]^{-1}$

4. $25 \bullet 5^{\frac{2}{3}}$

5. $x(x^2)^n$

6. $\frac{3x^2y^5}{9x^7y}$

7. $\left[\frac{2x^3y^4}{xy^7}\right]^2$

8. $125^{1/3}$

9. $\left[\frac{81}{100}\right]^{-3/2}$

10. $\sqrt[3]{\frac{-27r^6}{125s^9}}$

11. $-8^{\frac{2}{3}}$

12. $(-8)^{\frac{2}{3}}$

13. $(x+y)^3 \cdot (x+y)^4$

14. $(x^{-1} - y^{-1}) \div (x-y)^{-1}$

15. $\left(16^{\frac{2}{3}}\right)^{\frac{-3}{4}}$

16. $\left[\frac{16x^3y^{12}z^2}{2x^{-6}z^8}\right]^{\frac{2}{3}}$

Logarithms

Logarithms are the way to "undo" an exponent sentence, much like division undoes a multiplication problem. If one is good at exponents, it is easy to do logarithms.

Given the exponent sentence $2^3 = 8$, the "2" is called the *base*, the "3" is the *exponent* and the "8" is the *answer*. To rewrite this sentence as a logarithm, it becomes $\log_2 8 = 3$. This statement would be read: "log to the base 2 of 8 equals 3".
Note that the answer to a log sentence is ALWAYS the exponent.

Thus the **definition of a logarithm** is: $\log_b A = n$ if and only if $b^n = A$.

The base used for logs can vary, but the base is **always positive**. This means that the "argument", or A, is always positive. (Why?) The most commonly used bases are 10 and the natural logarithm, whose base is e ($e \simeq 2.718$).

Log to the base 10 is written $\log A = n$ by convention (note that the base is not written in this special logarithm) and translates to $10^n = A$. The natural logarithm is written $\ln A = n$ (this is the convention for abbreviating $\log_e A = n$) and translates to $e^n = A$.

Good to know:

$\log_b 1 = 0$ $\qquad\qquad$ $\log_b b = 1$

Try rewriting these as exponent statements to prove this for yourself.

Inverse Properties of logs:

$\log_b b^x = x$ $\qquad\qquad$ $b^{\log_b x} = x$

Laws of Logarithms:

1. $\log_b x + \log_b y = \log_b (x \bullet y)$

2. $\log_b x - \log_b y = \log_b \left(\dfrac{x}{y}\right)$

3. $n \bullet \log_b x = \log_b x^n$

<u>Logarithms: Practice Exercises</u>

Translate into log notation:

1. $2^7 = 128$ **2.** $9^{\frac{3}{2}} = 27$ **3.** $2^{-3} = \frac{1}{8}$

Translate into exponential notation:

4. $\log_3 9 = 2$ **5.** $\log_5 \frac{1}{5} = -1$ **6.** $\log_{35} 1 = 0$

Find N:

7. $\log_{10} N = 2$ **8.** $\log_5 N = -2$ **9.** $\log_{100} N = -\frac{7}{2}$

Solve for x:

10. $\log_8 64 = x$ **11.** $\log_2 (\frac{1}{32}) = x$ **12.** $\log_9 \frac{1}{27} = x$

Find a:

13. $\log_a 64 = 3$ **14.** $\log_a \frac{1}{7} = -2$ **15.** $\log_a 125 = \frac{3}{2}$

Given the log values below, find the numerical value of the following logarithms, using log properties. (Note these are not calculator problems):

log 2= 0.301	**log 3= 0.477**	**log 5= 0.699**	**log 7= 0.845**
ln 2= 0.693	**ln 3= 1.099**	**ln 5= 1.609**	**ln 7= 1.946**

16. $\log 14$ **17.** $\ln 8$ **18.** $\log \frac{5}{7}$

19. $\ln \sqrt{5}$ **20.** $\ln e^{100}$ **21.** $\log_5 5^{102}$

Write as the sum or difference of simpler log quantities:

22. $\ln \left(\frac{a^2 b^4}{z^3} \right)$

Express as a single log with leading coeficient 1:

23. $\frac{1}{2}(3\ln x + \ln y — \ln z)$

Solve the following log equations:

24. $\log 10 + \log 3 = \log x$ **25.** $\log(x+1) — \log(x-1) = \log 8$

26. $\log_2 x + \log_2(x+2) = 2$

Factoring

Factoring a polynomial means expressing it as a product. Why is this important?
First suppose you have the math problem: $a + b = 4$. There are infinite solutions to this problem. For example, a=6 and b=–2; or a=–3 & b=7; another possible solution is $a=\frac{1}{2}$ & $b=\frac{7}{2}$ and so on. Now try a different equation, this time involving multiplication instead of addition: $a \cdot b=0$. How many solutions are there to this problem? Only two: either a=0 or b=0. For this reason, we factor. Factoring can be viewed as a method for turning a polynomial into a product or multiplication problem.

Special Products and Factoring Techniques

Distributive law	***<u>Examples</u>***
1. $ax + ay = a(x + y)$	$4x + 2x^2 = 2x(2+x)$
Simple Trinomial	
2. $x^2 + (a+b)x + a \cdot b = (x + a)(x + b)$	$x^2 + 6x + 8 =$ $x^2 + (2+4)x + 2 \cdot 4 = (x + 2)(x + 4)$
Difference of Squares	
3. $x^2 - a^2 = (x-a)(x+a)$	$x^2 - 9 = (x-3)(x+3)$
4. $x^4 - a^4 = (x^2 - a^2)(x^2 + a^2)$ $= (x-a)(x+a)(x^2 + a^2)$	$x^4 - 16 = (x^2 - 4)(x^2 + 4)$ $= (x-2)(x+2)(x^2 + 4)$
Sum or Difference of Cubes	
4. $x^3 + a^3 = (x+a)(x^2 - ax + a^2)$	$x^3 + 8 = (x+2)(x^2 - 2x + 4)$
5. $x^3 - a^3 = (x-a)(x^2 + ax + a^2)$	$x^3 - 27 = (x-3)(x^2 + 3x + 9)$
Factoring by Grouping	
6. $acx^3 + adx^2 + bcx + bd = ax^2(cx+d) + b(cx+d)$ $= (ax^2 + b)(cx + d)$	$3x^3 - 2x^2 - 6x + 4 = x^2(3x-2) - 2(3x-2)$ $= (x^2 - 2)(3x-2)$
Quadratic Formula	
7. $ax^2 + bx + c = 0 \longrightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$x^2 + 3x - 1 = 0 \longrightarrow x = \dfrac{-3 \pm \sqrt{3^2 - 4(1)(-1)}}{2(1)}$ $= \dfrac{-3 \pm \sqrt{13}}{2}$

<u>Factoring: Practice Exercises</u>

Factor completely:

1. $2ax + 4a^2 x^2$

2. $x^3 + x^2 + x + 1$

3. $400 - 25y^2$

4. $(a+b)^2 - c^2$

5. $x^6 - 7x^3 - 8$

6. $x^2 + 100$

7. $16m^2 - 56mn + 49n^2$

8. $4^6 - 27x^3$

9. $x^2 + 3x + xw + 3w$

10. $1 - x^2 - 2xy - y^2$

11. $5x^5 + 30x^3 + 45x$

12. $2x^2 - 7xy + 6y^2$

13. $x^4 - 1$

14. $x^2 - 6$

15. $x^4 + 4x^2 - 45$

16. $2x^2 (x+3) + 4x(x+3)^2$

17. $(x^2 +1)^{-1/2}(x-7)^2 + (x^2 +1)^{1/2}(x-7)$

Algebraic Fractions

Some examples of algebraic fractions are: $\frac{2}{x}$, $\frac{x^2+2x-4}{x+6}$, and $\frac{1}{\sqrt{x^2+1}}$. Algebraic fractions that have a polynomial in both the numerator and denominator are called *rational expressions.* The first two examples are rational expressions.

Operations with Fractions

Operations	***Examples***
1. Add fractions (find a common denominator) $\frac{a}{b} + \frac{c}{d} = \frac{a}{b}\left(\frac{d}{d}\right) + \frac{c}{d}\left(\frac{b}{b}\right) = \frac{ad+bc}{bd}$	$\frac{2}{x} + \frac{3}{5} = \frac{2}{x}\left(\frac{5}{5}\right) + \frac{3}{5}\left(\frac{x}{x}\right) = \frac{10+3x}{5x}$
2. Subtract fractions (find a common denominator) $\frac{a}{b} - \frac{c}{d} = \frac{a}{b}\left(\frac{d}{d}\right) - \frac{c}{d}\left(\frac{b}{b}\right) = \frac{ad-bc}{bd}$	$\frac{2}{3} - \frac{x}{6} = \frac{2}{3}\left(\frac{2}{2}\right) - \frac{x}{6}\left(\frac{1}{1}\right) = \frac{4-x}{6}$
3. Multiply fractions $\left(\frac{a}{b}\right)\left(\frac{c}{d}\right) = \frac{ac}{bd}$	$\left(\frac{4}{x}\right)\left(\frac{3}{5}\right) = \frac{12}{5x}$
4. Divide fractions	
i) invert and multiply $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \bullet \frac{d}{c} = \frac{ad}{bc}$	$\frac{\frac{3}{x}}{\frac{4}{7}} = \frac{3}{x} \bullet \frac{7}{4} = \frac{21}{4x}$
ii) (multiply by common denominator) $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{\frac{a}{b}}{\frac{c}{d}}\left(\frac{\frac{bd}{1}}{\frac{bd}{1}}\right) = \frac{ad}{bc}$	$\frac{\frac{4}{2+x}}{\frac{3}{5-x}} = \frac{\frac{4}{2+x}}{\frac{3}{5-x}}\left(\frac{\frac{(2+x)(5-x)}{1}}{\frac{(2+x)(5-x)}{1}}\right) = \frac{4(5-x)}{3(2+x)}$

5. Cancel

$$\frac{ab}{ad} = \frac{b}{d} \qquad \frac{3x}{3y} = \frac{x}{y}$$

$$\frac{ab+ac}{ad} = \frac{a(b+c)}{ad} = \frac{b+c}{d} \qquad \frac{4x+4y}{4z} = \frac{4(x+y)}{4z} = \frac{x+y}{z}$$

6. Rationalizing

a. If the numerator or denominator is $\sqrt{a}$, multiply by $\frac{\sqrt{a}}{\sqrt{a}}$

b. If the numerator or denominator is $\sqrt{a} - \sqrt{b}$, multiply by $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}}$

c. If the numerator or denominator is $\sqrt{a} + \sqrt{b}$, multiply by $\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}-\sqrt{b}}$

Algebraic Fractions: Practice Exercises

Simpify as completely as possible:

1. $\frac{3x^2 - 75y^2}{3x^2 - 21xy + 30y^2}$ **2.** $\frac{a^2 - b^2}{b - a}$ **3.** $\frac{4-2x}{6x+30} \bullet \frac{x^2-25}{x^2-7x+10}$

4. $\dfrac{\dfrac{5x^2-9x-2}{30x^3+6x^2}}{\dfrac{x^4-3x^2-4}{2x^8+6x^7+4x^6}}$ **5.** $\left(1-\frac{x}{y}\right) \div \left(y - \frac{x^2}{y}\right)$ **6.** $\frac{5}{x+2} + \frac{5}{2-x} - \frac{6}{x^2-4}$

7. $\frac{3x-4}{x^2-9} - \frac{2x-3}{x^2-x-6}$ **8.** $\dfrac{\frac{1}{x} - \frac{1}{3}}{x - \frac{9}{x}}$ **9.** $\dfrac{1 - \frac{4}{x^2-y^2}}{1 + \frac{2}{x+y}}$

10. Rationalize: $\frac{6}{\sqrt{7} - \sqrt{3}}$

Equations

Most of the time we are asked to solve equations in math. This means to find all the *solutions* or *roots*. The solutions of an equation are not affected as long as you add,subtract, multiply or divide both sides of the equation by the same quantity. The only exception is dividing by zero.

The method of solving an equation depends on the *degree* of the equation. *First degree equations* are solved using addition, subtraction, multiplication and division.

ex. $3x + 7 = 9$

$3x = 2$ Subtract 7 from both sides.

$\frac{3x}{3} = \frac{2}{3}$ Divide both sides by 3.

$x = \frac{2}{3}$

Second degree equations or *quadratic equations* are solved by factoring or the quadratic formula.

ex. $x^2 - 7x - 8 = 0$

Factoring

$(x-8)(x+1)=0 \longrightarrow x = 8 \text{ or } x = -1$

Quadratic formula

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(1)(-8)}}{2} = \frac{7 \pm \sqrt{81}}{2}$$

$$= \frac{7 \pm 9}{2} \longrightarrow x = 8 \text{ or } x = -1$$

Equations involving *absolute value* are equivalent to two equations without the absolute value sign.

ex. $|x+3| = 7 \longrightarrow +(x+3) = 7$ **or** $-(x+3) = 7$

$x+3 = 7$ $-x-3 = 7$

$x = 4$ $-x = 10$

$x = -10$

<u>Equations: Practice Exercises</u>

Solve for x:

1. $3(x - 4) + 5(x+6) = 14$

2. $\frac{x-4}{2} - \frac{x}{5} = \frac{1}{10}$

3. $\frac{4}{x+1} = \frac{3}{x} + \frac{1}{15}$

4. $\frac{6-x}{x^2-4} - 2 = \frac{x}{x+2}$

5. $x^2 - 8x = -10$

6. $3x^2 - x - 2 = 0$

Inequalities

An inequality is the mathematical expression of comparison:

The sentence	is stated mathematically
x is more than 2	$\mathbf{x > 2}$
a is between negative one and 5.	$\mathbf{-1 \le a < 5}$

Be sure to understand and read the symbols correctly:

The symbol	is read:
$>$	***greater than***
$<$	***less than***
$\ge$	***greater than OR equal to***
$\le$	***less than OR equal to***

An important distinction to remember about algebraic inequalities is that, generally, they have an **infinite** number of real solutions. An algebraic equation has one, two or more **finite** real solutions.

SOLVING LINEAR INEQUALITIES

Linear inequalities are solved in a similar manner to solving linear equations, with one important difference. Like equations, any value added or subtracted from one side of the inequality must also be equivalently added or subtracted from the other side. The same is true for multiplying and dividing both sides by posititve numbers. However, **multiplying or dividing both sides of an inequality by negative numbers requires the inequality sign to be reversed.**

Ex 1. $-x - 10 < 20 + x$

$-x < 30 + x$ *add10 to both sides*

$-2x < 30$ *subtract x from both sides*

$x > -15$ *divide by –2,* ***reverse sign.***

SOLVING ABSOLUTE VALUE INEQUALITIES

Just as with absolute value equations, you must consider two cases. The value inside the absolute value must have the correct magnitude, but it may be either positive or negative. If this value is negative you must make it positive by multiplying by –1. **With absolute value inequalities, you will always have two problems to solve.**

Ex 2. $|x - 5| \ge 7$

Case I: (x – 5) is positive

$(x - 5) \ge 7$

$x \ge 12$

Case II: (x – 5) is neg, make it positive

$-(x - 5) \ge 7$

$(x - 5) \le -7$

$x \le -2$

SOLVING HIGHER ORDER INEQUALITIES

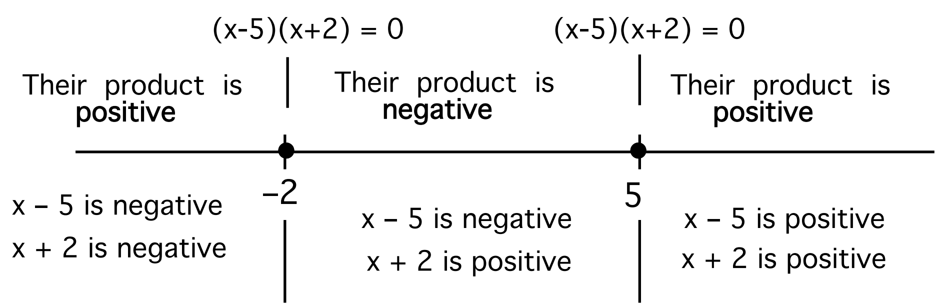
Higher order inequalities **cannot be solved in the same way as higher order equalities.** You must be able to account for all possible combinations of multiplying positive and negative terms. While there is more than one method for solving these inequalities, using a number line is the most efficient and effective method. The following example illustrates this method.

Ex 3. $x^2 - 3x - 10 \geq 0$

$(x - 5)(x + 2) \geq 0$ *factored form*

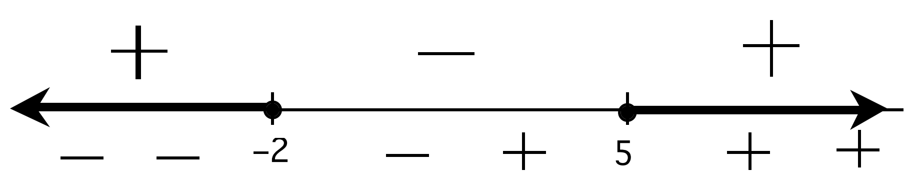
$x = -2, 5$ *roots (where each term = 0)*

Place these roots on a number line, using **open circles** for > or <, and **closed circles** for ≥ or ≤.



Test each region to determine the sign of each term.

This information can be more easily represented in symbol form as follows:



The inequality sign determines which region is the solution. Here, ≥ 0means the positive regions

This number line method works well for any higher power inequality with many factored terms or rational (division) inequalities.

Caution: Problems which are rational (like Practice exercises 7 and 8) may have both open **and** closed circles. Even if you have and inequality which includes "equals" (≤ or ≥), you must never use a closed circle on a root which causes you to divide by zero!

Inequalities: Practice Exercises

Solve the following inequalities. Give your answer in both inequality and interval notation.

1. $3x + 2 \geq 0$

2. $7 - 4x < 5$

3. $|2x + 1| < 2$

4. $|4 - 3x| \geq 5$

5. $x^2 + 2x - 3 < 0$

6. $x^2 \geq 4$

7. $(3 - x)(4 - x) > 0$

8. $x^2 - 2x + 1 \geq 0$

9. $x^2 - 7x \leq -6$

10. $(x - 4)^3(3 - x)(5 - x)^2 > 0$

11. $\dfrac{(x - 3)(x + 1)}{(x - 2)(x - 1)} > 0$

12. $\dfrac{x^4 - 16}{x^2 - 4x - 5} \leq 0$

13. $(x + 4)^5(x - 3)^3 + (x + 4)^4(x - 3)^4 < 0$

14. $(x - 5)(x + 2)^7 - (x - 5)^2(x + 2)^6 \geq 0$

Graphing Techniques

The best place to start on any graph is to plot x- and y-intercepts(if any). For some graphs, like lines, this is enough information to complete your graph. In other cases there are a few other important characteristics - a vertex, center or possibly one or more asymptotes - which must be determined as well. An important graphing skill is recognizing the form of equation each general shape has. Completing the handout
"Graphs You Should Know" will help you familiarize yourself with the most common graphs.

Lines $Ax + By + C = 0$

Exponents on x and y are both one.

As stated above, you need only to find the y-intercept (let $x = 0$ and solve for y) and the x-intercept (let $y = 0$ and solve for x) and draw a line through them.

Absolute Values

$$y - k = |x - h|$$

Exponents on x and y are both one.
Absolute value signs around the x-term.
(If they are around the y-term it is "sideways" and not a function.)

Once you have found your intercepts you must graph its vertex. This is simply the point **(h,k)**, but you must be sure your equation is in the form given above.

Parabolas $\mathbf{y = ax^2 + bx + c}$

<u>One</u> variable is squared
(Note: $x = y^2$ is a "sideways" parabola, but <u>not</u> a function.)

After finding the x- and y-intercepts, you must also find the **vertex.** The x-coordinate of the vertex, V_x , may be found three ways:

1. $V_x = \frac{x_1 + x_2}{2}$ — Midpoint of the x-intercepts: x_1 & x_2 .

2. $V_x = \frac{-b}{2a}$ — a & b are constants from the std. form

With these first two methods the y-coordinate of the vertex, V_y must be found by plugging V_x back into the original formula: $\mathbf{V_y = a(V_x) + b(V_x) + c}$. The last method obtains both V_x and V_y :

3. $y - V_y = (x - V_x)^2$ — Complete the square

Circles $\mathbf{(x - h)^2 + (y - k)^2 = r^2}$

Both the x- and y-terms are squared
Note: Circles are <u>not</u> functions!

In addition to the x- and y-intercepts, you must plot the center **(h,k)** and demonstrate the length of the radius,r. If your equation is not in this form you must **complete the square** to put it in the correct form.

Translations

There are other functions which are easily graphed by translating (shifting) the standard shape up or down and/or left or right. Here is a list of some of thesefunctions:

FUNCTION	EXAMPLE
Cubic	$y = x^3$
Rational	$y = \frac{1}{x}$
Exponential	$y = 3^x$
Logarithmic	$y = \log_2(x)$
Trigonometric	$y = \sin x$ $y = \cos x$ $y = \tan x$

The rules of translating any basic function are as follows:

a. $y = f(x) + \mathbf{C}$ is moved **up** C units from $y = f(x)$.
b. $y = f(x) - \mathbf{C}$ is moved **down** C units from $y = f(x)$.

c. $y = f(x - \mathbf{C})$ is moved to the **right** C units from $y = f(x)$.
d. $y = f(x + \mathbf{C})$ is moved to the **left** C units from $y = f(x)$.

<u>Graphing Practice Exercises</u>

Graph the following equations.

1. $y = 2x + 1$

2. $y = (x - 2)^2$

3. $3x - 4y = 12$

4. $y = |x - 4|$

5. $y = \frac{-4}{x}$

6. $y = (x + 1)^3$

7. $y = -3$

8. $y = |x| + 3$

9. $y = 3^x$

10. $y = -x + 1$

11. $y = \frac{9}{(x-2)}$

12. $y = 10^x$

13. $y = \log_4(x) - 1$

14. $y = x^2 - 6x + 3$

15. $y = -x^2 + 4x$

16. $y = x^3 - 2$

17. $y = \sqrt{x}$

18. $x = -1$

Functions

A function is a relationship between two variables where each value of the independent variable only corresponds exactly to one value of the dependent variable . In layman's terms, every x has only one y. Another way to look at it is, if you could list all the ordered pairs, no two unique ordered pairs have the same first component. Graphically, a function passes the "vertical line test"; any vertical line hits the graph only once.

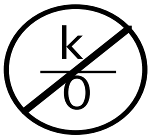
Some examples of functions are

$P(x) = 3x^3 + 4x^2$	*Polynomial function*
$f(x) = -2$	*Constant function*
$g(x) = ax + by + c$	*Linear function*
$h(x) = ax^2 + bx + c$	*Quadratic function*
$R(x) = \frac{P(x)}{Q(x)}$	*Rational function,* P(x) & Q(x) are functions, $Q(x) \neq 0$
$f(x) = e^x$	*Exponential function*
$g(x) = \ln(x)$	*Logarithmic function*

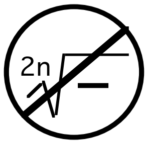
Domain

The domain of a function is all of the values of the independent variable **for which its function is defined.** Informally many people think of the domain as all of the possible first components of all the ordered pairs, or all the "x-values" that "work" in the function.

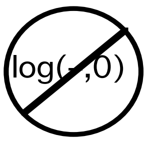
In finding the domain of any function, you need to consider three rules:



You cannot divide by zero
D $\neq$ 0



You cannot take the even root of a negative number
R $\geq$ 0



You cannot take the logarithm of a negative or of zero
L > 0

Range

The range of a function is all of the values of the dependent variable. Again informally, you may think of the range as all the second components of all the ordered pairs, or all the "y-values" which are the result of all the domain values.

There are three ways to determine the range of a function. Which method is best depends on both the individual problem and your personal preference.

1. **Graph.** You can visually determine all of the possible y-values if you can graph the function.
2. **Analysis.** Sometimes you will need to ask yourself some questions about the values of y. However, you will need to **find the domain** first, in order to try various values of x to answer the following questions.

   a. Can $y = 0$?
   b. Can $y < 0$? (be negative?) How far (to $-\infty$)?
   c. Can $y > 0$? (be positive?) How far (to $+\infty$)?
3. **Inverse.** For functions that are **one-to-one**, that is, they have the addtional property of having only one x-value for every y-value, the **range of f(x) is the domain of the inverse function, $\mathbf{f^{-1}(x)}$.**

Composition of a function

The composition of a function is a way of combining functions. There are two types of notation:

$$(f \circ g)(x) = f(g(x))$$

Both notations are directing you to first find g(x). The **result** of g(x) is then put in to the function f(x). Note: **Order does matter!** Except for some special cases (see inverses below) generally $f(g(x))$ **is not equivalent to** $g(f(x))$.

Inverses

Two functions, f(x) and g(x), are inverses if the components of every ordered pair of f(x) are in interchanged positions in the function g(x) and vice versa. That is, all the x-values in f(x) are y-values in g(x) and all the y-values in f(x) are x-values in g(x).

To find the inverse of f(x), substitue y for f(x) in your function, then interchange all variables: all x's become y's and all y's become x's. Solve for y. A common notation for the **inverse of f(x) is $f^{-1}(x)$** . Be careful! When used in this way with functions this notation never means "reciprocal". It always means "inverse function".

To prove two functions, f(x) and g(x), are inverses you must show that for every x in the domain of g(x), **f(g(x)) = x**, and for every x in the domain of f(x), **g(f(x)) = x.**

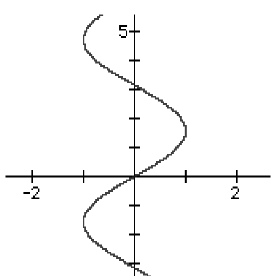
Functions: Practice Exercises

1. Let $f(x) = x^2 - x$. Find:

a. f(0) **b.** f(–1) **c.** f(1) **d.** f(x + h)

2.

a.



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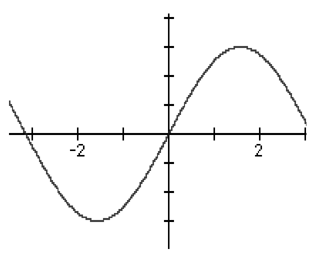
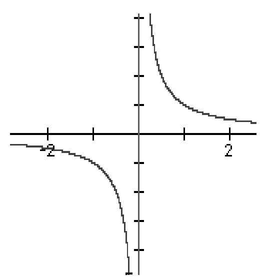
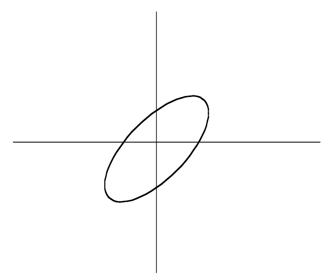


c.



d.



a. Which of the graphs is a function?
b. For the one(s) which are function(s), give the domain and range.

3. Find the domain of each of the given functions:

a. $f(x) = \sqrt{4 - x^2}$ **b.** $g(x) = \dfrac{1}{x^2 - x - 2}$

c. $y = \sqrt[3]{x^2 + 3x - 4}$ **d.** $y = \dfrac{2x + 4}{\sqrt{x^2 + 5x + 4}}$

4. Find the range of each of the given functions:

a. $y = \dfrac{1}{x}$ **b.** $f(x) = \sqrt{x - 3}$ **c.** $y = 3x^2 - 2$

5. For each of the following functions, simplify $\dfrac{f(x + h) - f(x)}{h}$.

a. $f(x) = x^2 - 3x + 5$ **b.** $f(x) = \sqrt{x + 2}$ **c.** $f(x) = \dfrac{3}{7 - x}$

6. Let $f(x) = \frac{1}{3}x - 5$ and $g(x) = \dfrac{1}{x + 1}$.

a. Find $f^{-1}(x)$ **b.** Find $g^{-1}(x)$

7. Using the functions given in Problem 6, find $f(g(x))$ and $g(f(x))$.

Practice Exercise Answers

Exponents and Radicals:

1. 3
2. $—8x^6 y^9$
3. 49
4. $5^{\frac{8}{3}}$
5. x^{2n+1}
6. $\frac{y^4}{3x^5}$
7. $\frac{4x^4}{y^6}$
8. 5
9. $\frac{1000}{729}$
10. $\frac{-3r^2}{5s^3}$
11. -4
12. 4
13. $(x+y)^7$
14. $\left(\frac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x-y}}\right) = \frac{(x-y)(y-x)}{xy}$
15. $\frac{1}{4}$
16. $\frac{4x^6 y^8}{z^4}$

Logarithms:

1. $\log_2 128 = 7$

2. $\log_9 27 = \frac{3}{2}$

3. $\log_2\left(\frac{1}{8}\right) = -3$

4. $3^2 = 9$

5. $5^{-1} = \frac{1}{5}$

6. $35^0 = 1$

7. $N = 100$

8. $N = \frac{1}{25}$

9. $N = \frac{1}{10^7}$

10. $x = 2$

11. $x = -5$

12. $x = -\frac{3}{2}$

13. $a = 4$

14. $a = \sqrt{7}$

15. $a = 25$

16. 1.146

17. 2.079

18. -0.146

19. 0.805

20. 100

21. 102

22. $2\ln a + 4\ln b - 3\ln z$

23. $\ln\left(\sqrt{\frac{x^3 y}{z}}\right)$

24. $x = 30$

25. $x = \frac{9}{7}$

26. $-1 + \sqrt{5}$

Factoring:

1. $2ax(1+2ax)$

2. $x^2 (x+1) + 1(x+1) = (x^2 +1)(x+1)$

3. $25(16 - y^2) = 25(4-y)(4+y)$

4. $[(a+b) + c][(a+b) - c]$

5. $(x^3)^2 - 7x^3 - 8 = (x^3 - 8)(x^3 + 1)$
$= (x-2)(x^2 + 2x + 4)(x + 1)(x^2 - x + 1)$

6. does not factor

7. $(4m - 7n)^2$

8. $(4^2)^3 - (3x)^3$
$=(4^2 - 3x)[(4^2)^2 + 48x + (3x)^2]$
$= (16-3x)(256 + 48x + 9x^2)$

9. $x(x+3) + w(x+3) = (x+w)(x+3)$

10. $1-(x^2 + 2xy + y^2) = 1-(x+y)^2$
$= [1-(x+y)][1+(x+y)]$

11. $5x(x^4 + 6x^2 + 9) = 5x(x^2 + 3)^2$

12. $(2x - 3y)(x - 2y)$

13. $(x^2 + 1)(x^2 - 1) = (x^2 + 1)(x+1)(x-1)$

14. $(x - \sqrt{6})(x + \sqrt{6})$

15. $(x^2 + 9)(x^2 - 5)$

16. $2x(x+3)[x+2(x+3)]$
$=2x(x+3)(3x+6)$
$= 2x(x+3)3(x+2)$
$= 6x(x+3)(x+2)$

17. $(x^2 + 1)^{-1/2}(x-7)\left[(x-7)+(x^2 + 1)\right] = (x^2 + 1)^{-1/2}(x-7)\left[x^2 + x - 6\right]$
$= (x^2 + 1)^{-1/2}(x-7)(x + 3)(x-2)$

Algebraic Fractions

1. $\dfrac{x+5y}{x-2y}$ **2.** $-(a+b)$ or $-a-b$ **3.** $-\dfrac{1}{3}$ **4.** $\dfrac{x^4(x+1)}{3(x^2+1)}$

5. $\dfrac{1}{x+y}$ **6.** $\dfrac{26}{4-x^2}$ **7.** $\dfrac{x^2-x+1}{(x+3)(x-3)(x+2)}$ **8.** $-\dfrac{1}{3(x+3)}$

9. $\dfrac{x^2 - y^2 - 4}{x^2 - y^2 + 2x - 2y}$ **10.** $\dfrac{3(\sqrt{7} + \sqrt{3})}{2}$

Equations

1. $x = \frac{-1}{2}$ **2.** $x = 7$ **3.** $x = 5,9$ **4.** $x = \frac{7}{3}, -2$

5. $x = 4 \pm \sqrt{6}$ **6.** $x = 1, \frac{-2}{3}$

Inequalities

1. $x \geq \frac{-2}{3}$; $[\frac{-2}{3}, \infty)$

2. $x > \frac{1}{2}$; $[\frac{1}{2}, \infty)$

3. $\frac{-3}{2} < x < \frac{1}{2}$; $(\frac{-3}{2}, \frac{1}{2})$

4. $x \le \frac{-1}{3}$ OR $x \ge 3$; $(-\infty, \frac{-1}{3}] \cup [3, \infty)$

5. $-3 < x < 1$; $(-3, 1)$

6. $x \le -2$ OR $x \ge 2$; $(-\infty, -2] \cup [2, \infty)$

7. $x < 3$ OR $x > 4$; $(-\infty, 3) \cup (4, \infty)$

8. All real numbers ; $(-\infty, \infty)$

9. $1 \le x \le 6$; $[1,6]$

10. $x < 3$ OR $4 < x < 5$ OR $x > 5$;
$(-\infty, 3)$ U $(4, 5) \cup (5, \infty)$

11. $x < -1$ OR $1 < x < 2$ OR $x > 3$
$(-\infty, -1) \cup (1, 2) \cup (3, \infty)$

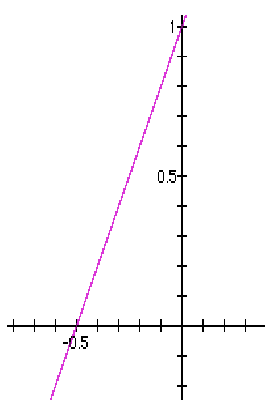
12. $-2 \le x < 0$ OR $2 \ge x > 5$;
$[-2, -1) \cup [2, 5)$

13. $\frac{1}{2} < x < 3$; $(-\frac{1}{2}, 3)$

14. $x = -2$ OR $x \ge 5$; $-2 \cup [5, \infty)$

Graphing

1.



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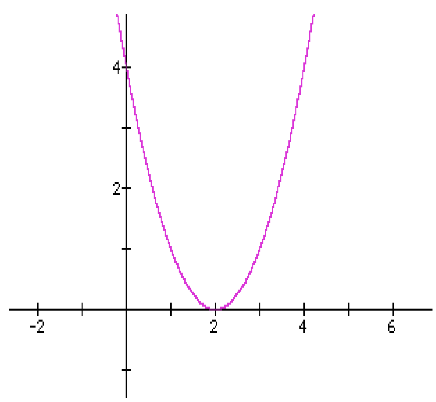

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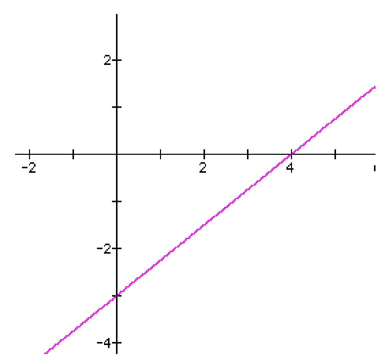

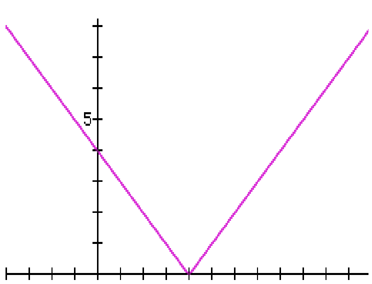

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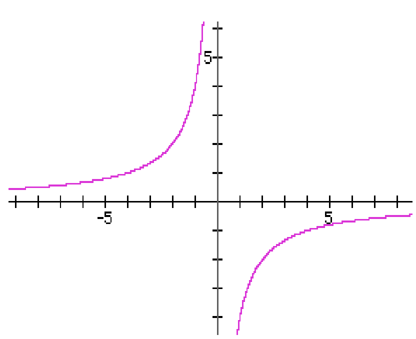

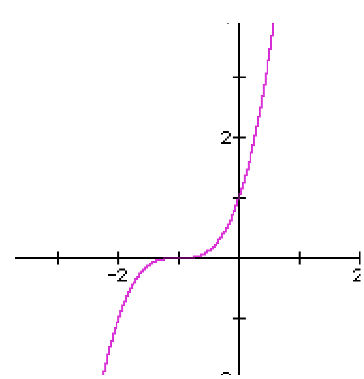

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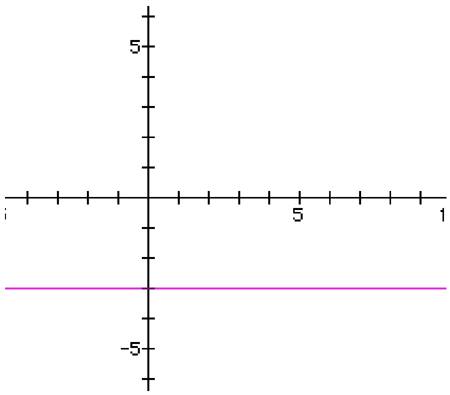

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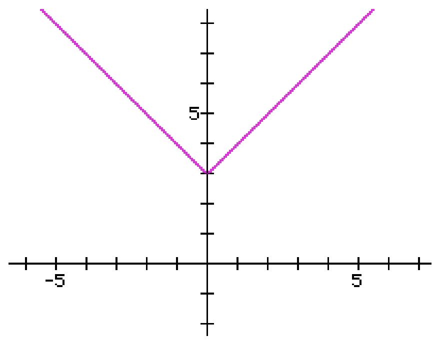

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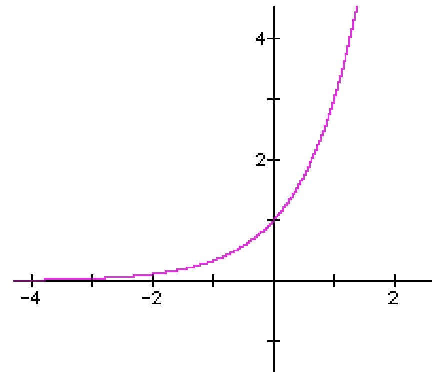

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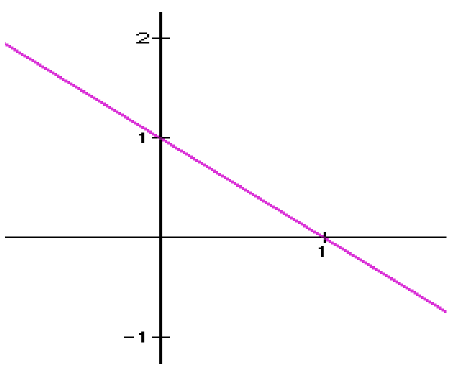

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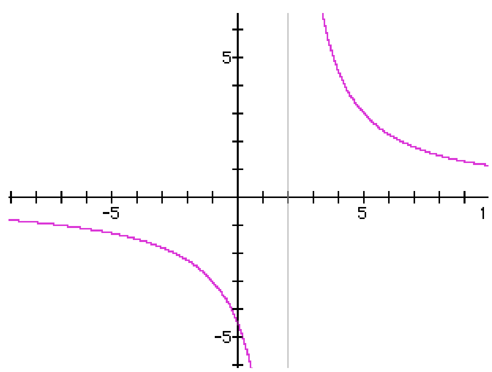

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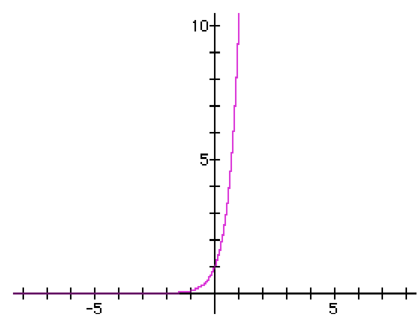

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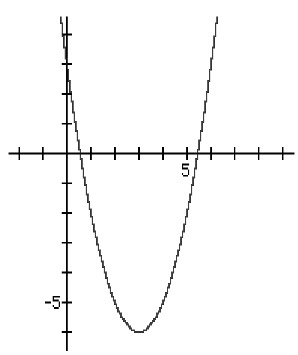

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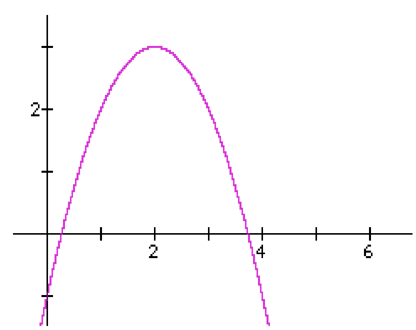
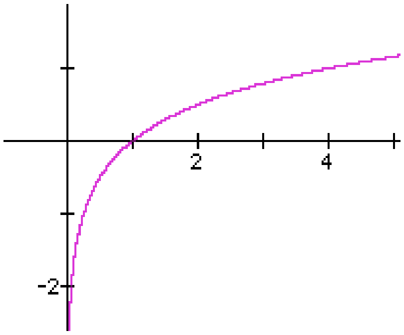
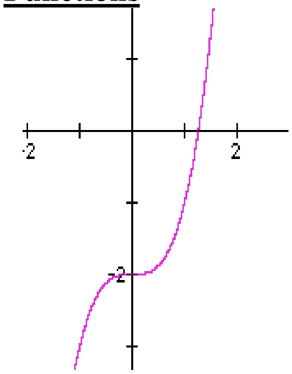




16.

<u>Functions</u>



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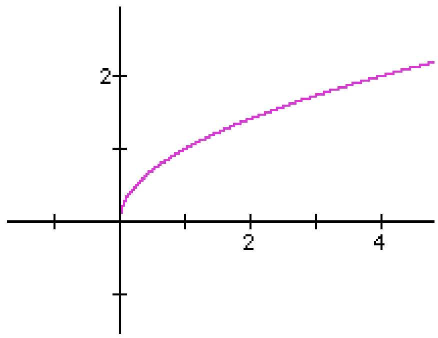

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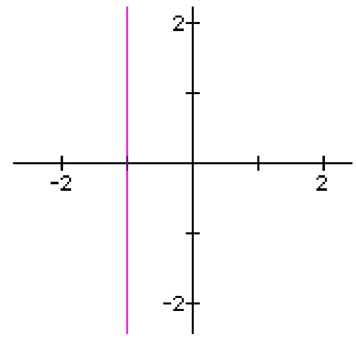

1. a. 0 **b.** 2 **c.** 0 **d.** $(x + h)^2 - (x + h) = x^2 + 2xh + h^2 - x - h$

2. **a.** Graphs II and III
b. For functionII: Domain is all real numbers, Range is $-2 \leq y \leq 1$
For function III: Domain is $x \neq 0$, and Range is $y \neq 0$.

3. The domain of each of the given functions is:
a. $[-2,2]$ **b.** $x \neq -1$, or 2 **c.** All real numbers **d.** $(-\infty, -4) \cup (-1, \infty)$

4. The range of each of the given functions is:
a. $y \neq 0$ **b.** $y \geq 0$ x – 3) **c.** $y \geq -2$

5. For each of the given functions, the simplification of $\dfrac{f(x + h) - f(x)}{h}$ is:

a. $2x + h - 3$ **b.** $\dfrac{1}{\sqrt{x+h+2} + \sqrt{x+2}}$ **c.** $\dfrac{3}{[7-(x+h)](7 - x)}$

6. a. $f^{-1}(x) = 3x + 15$ **b.** $g^{-1}(x) = \dfrac{1 - x}{x}$

7. $f(g(x)) = \dfrac{1}{3} \bullet \dfrac{1}{x+5} - 5$, $g(f(x)) = \dfrac{1}{(\frac{1}{3}x - 5)} + 1 = \dfrac{1}{3x + 15} - 5 = \dfrac{3}{x - 12}$