

MAT280 - COUNTING CURVES ON SURFACES - SPRING 2022

1. INTRODUCTION

The goal of this course will be to study the problem of counting curves on a surface: given a surface, how many curves of at most a given length does it have? These types of problems have a rich history starting from work of Huber and Margulis in the sixties, until recent work of Mirzakhani and Erlandsson-Souto. The goal of this course is to introduce the basics of the geometry of surfaces and curves, study some curve counting problems in the literature, and present the recent proof of a counting result by Erlandsson and Souto. In this quest, we will introduce several objects such as the Bers-Teichmüller space, the mapping class group, measured laminations and train tracks, and a key overarching piece of machinery: geodesic currents. Although this is mostly a low dimensional geometry and topology course, some of the tools we will use, such as geodesic currents, might also be of interest to people interested in geometric analysis. The problems of curve counting have connections to several other areas of mathematics, including combinatorics and number theory.



2. LOGISTICS

Instructor: Didac Martinez Granado - dmartinezgranado@ucdavis.edu

Lectures: Time: Monday, Wednesday, Friday 10-10.50pm. **Venue:** Social Science Humanities 80

Course website: <https://sites.google.com/iu.edu/didac-martinez-granado/teaching/mat-280-counting-curves-on-surfaces>

Contact: Fastest way to contact is email or Piazza. For questions about the material, asking on Piazza is preferable. For more personal questions (e.g. discussing grades) use my email address: dmartinezgranado@ucdavis.edu. I check both very often. I encourage you to come to office hours or to email me to set alternate appointments.

Platforms: I will use my personal website for the material of the course, Canvas to administer grades, as well as Piazza for questions and discussions.

Regular office hours: Monday, Wednesday, Friday 11-12pm. I am also happy to set alternate appointments.

Textbook: We will use a variety of sources, some of them indicated below, and in more detail on the Course Website. I encourage you to take your own notes, although notes will be provided, too.

Grade :

- (1) **(50%)** Homework Weekly homework consisting of about 5 problems, due Mondays, in class.
- (2) **(20%)** Exam Midterm exam, 50 minutes, 4 problems.
- (3) **(30%)** Final project: report and presentation.

3. TENTATIVE CONTENT

These are some snippets of the content we will see. A more detailed calendar will appear soon on the Course Website.

- (1) Basics of geometry of surfaces, and hyperbolic geometry
- (2) Curves on surfaces, simple vs non-simple curves
- (3) Teichmüller space
- (4) Mapping class group
- (5) Train-tracks, measured laminations
- (6) Geodesic currents
- (7) Overview on curve counting results: for simple curves, for non-simple, for fixed type.
- (8) Proof of the fixed type problem: radallas, smoothings
- (9) Applications: curve functionals and extension to geodesic currents

4. RECOMMENDED RESOURCES:

We will be using material from the following recommended sources.

Books:

- Pending publication we will use the following book as a main source, and especially for point 9: **Mirzakhani's counting results and geodesic currents**, Viveka Erlandsson and Juan Souto, (to be published by Springer Birkhäuser 2022).
- This will be a good source for points 1-4: **Primer on mapping class groups**, Benson Farb and Farb Margalit, Princeton Mathematical Series
- This will be a good source for point 5: **Combinatorics of train-tracks**, Harer and Penner, Princeton University Press
- This will be a good source for points 1-2: **Geometry and Spectra of Compact Riemann surfaces**, Peter Buser, Modern Birkhäuser Classics
- This will be a good source for point 5: **Hyperbolic manifolds and discrete groups**, Michael Kapovich, Modern Birkhäuser Classics

Notes:

- These are unpublished notes, but will also be useful for points 2-3 specially, and 5 as well. They are available at the author's website: **Closed curves on surfaces**, Francis Bonahon
- These are published notes, which will be useful for points 6 and 7: **Hyperbolic Structures on Surfaces and Geodesic Currents**, Javier Aramayona and Christopher Leininger
- These are published notes, which will be useful for points 1-7: **Degenerations of hyperbolic structures on surfaces**, Christopher Leininger

Papers:

- This will be used for point 9: **Counting curves in hyperbolic surfaces**, Viveka Erlandsson and Juan Souto, Geometric Analysis and Functional Analysis
- This will be used for point 8: **Growth of the number of simple closed geodesics on hyperbolic surfaces**, Maryam Mirzakhani, Annals of Math
- This preprint will be used for point 8 and 9 **Counting Mapping Class group orbits on hyperbolic surfaces**, Maryam Mirzakhani, preprint
- This paper will be used for point 8 and 9 **Geodesic currents and counting problems**, Kasra Rafi and Juan Souto, Geometric Analysis and Functional Analysis
- This paper will be used for point 9 and 10 **From Curves to currents**, Didac Martinez-Granado and Dylan Thurston, Forum of Mathematics, Sigma

Problem sets: Problem sets are due **Mondays at the beginning of class**. You will be allowed one and only one unexcused extension, provided you communicate it to me before the deadline.

Midterm: There will be a midterm, date TBA. It will be during class, last 50 minutes, and consist of 4 problems.

Final project: A list of final projects will be made available on the Course website. Each student will have to choose one topic before the deadline (TBA) and write a 5 page report as well as deliver a short 15 minute presentation.

Special accommodations: If you require special accommodations, please let me know in advance, by contacting the Student Disability Center (SDC). "Any student with a documented disability (e.g. physical, learning, psychiatric, vision, hearing, etc.) who needs to arrange reasonable accommodations must contact the Student Disability Center (SDC). Faculty are authorized to provide only the accommodations requested by the SDC. If you have any questions, please contact the SDC at 530-752-3184 or sdc@ucdavis.edu."