

MATH 180: MANIFOLDS (WINTER 2023)

This course is an introduction to modern topology, the study of spaces through global properties that do not change when the space is stretched or pulled. The types of spaces we will study are manifolds, spaces which look like flat Euclidean space if you zoom in close enough. Manifolds can have any number of dimensions, but we will focus on manifolds of dimensions 2 and 3, and hint towards dimension 4. We will learn methods from modern low-dimensional topology which can be used to construct, study, and distinguish different manifolds. A great deal of the material will be very visual, and the goal is for students to learn methods to understand and visualize pieces of spaces which cannot be fully visualized in their entirety from our three-dimensional perspective. By the end of the course, students will begin to access topics that are used in active current research.

Prerequisites: Students are strongly recommended to have taken at least one course in geometry or topology such as MAT147, MAT141, or MAT 116, but the main requirement is hard work and enthusiasm towards stretching your visual and mathematical mind.

Students who have not taken MAT147 will need to work extra hard in the first week to learn a few key concepts in topology (continuous maps, homeomorphisms, open cover, subspace, quotient space, product of two spaces).

Course Topics: Here are examples of topics we might cover.

- Manifolds and dimension
- Spheres
- Tori and products
- Connected sum and other cut and paste
- Surfaces, polygonal presentations, Euler characteristic
- Coverings and branched coverings
- Handle decompositions
- Submanifolds, knots, surfaces
- Heegaard splittings and diagrams
- Trisections and Kirby diagrams

Course Grade: This will be an **interactive** class. Your grade will largely be a reflection of how much you engage with the material as measured in the following ways.

- 20% **Quizzes:** There will be 3-5 short quizzes.
- 30% **Exercises:** There will be weekly problem sets.
- 20% **Journal:** Students will be asked to maintain a written account of their understanding of the assigned readings and the class discussions. All students must have journal entries every week.
- 10% **Discussion:** Students will receive credit by coming to class with questions on assigned readings, asking questions during lectures, answering questions in class, and engaging in class discussions.
- 20% **Presentations:** Each student will be a part of a group that will study one topic related to the course in greater depth. The group will create a final written report and a presentation that could be presented live or as a video recording. Each student must have a clearly defined individual contribution to the project and grades will be assigned individually.