Name: ____________________________

Student ID: ________________________

Final Exam

Instructions: You absolutely must show your work! A correct answer without a reasonable amount of work will be given a 0. You are allowed 1 page of notes. Calculators and other outside information sources are not allowed during the exam. You do not need to simplify your final answers. Please leave your final answers in the boxes at the bottom of each page.

Problem 1. (5 points) Find the interval of convergence for the following series:

$$\sum_{n=1}^{\infty} \frac{(2x-1)^n}{(n^3)(5^n)}$$

Problem 2. (5 points) Write down the Taylor series centered at $x = 0$ for the following function:

$$f(x) = \frac{1}{(1-x)^2}$$

Problem 3. (8 points) Find the following limits:

a)

$$\lim_{(x,y)\to(2,-4)} \frac{y+4}{x^2y - xy + 4x^2 - 4x}$$

b)

$$\lim_{(x,y)\to(2,0)} \frac{\sqrt{2x-y}-2}{2x-y-4}$$

Problem 4. (5 points) Let

$$z = x^2y^2 + xy^3 + ye^{xy}, \quad x = r\cos(\theta), \quad \text{and} \quad y = r\sin(\theta).$$

Find

$$\left.\frac{\partial z}{\partial \theta}\right|_{(r,\theta)=(1,\pi)}.$$

Problem 5. (5 points) Find the directional derivative of the function

$$f(x, y) = \frac{x - y}{xy + 2}$$

at the point $(1, -1)$ in the direction of $\mathbf{v} = 12\mathbf{i} + 5\mathbf{j}$.

Problem 6. (5 points) Find an equation for the plane that is tangent to the surface determined by

$$z = e^{-(x^2+y^2)}$$

at the point $(0, 0, 1)$.

Problem 7. (8 points) Find all critical points of the function

$$f(x,y) = 2x^3 + 2y^3 - 9x^2 + 3y^2 - 12y$$

and identify them as local maxima, local minima, or saddle points.

Problem 8. (9 points) Find the absolute maximum and minimum of the function

$$f(x, y) = 4x - 8xy + 2y + 1$$

on the triangular plate bounded by the lines $x = 0$, $y = 0$, and $x + y = 1$ in the first quadrant.