MAT 280: Morse and Cerf theory in low dimensions

Term: Spring 2023
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Course Content

The goal of the course is to use Morse and Cerf theory to understand surfaces, then bootstrap this understanding to 3- and 4-manifolds. The class will begin with an introduction to spaces of smooth mappings. After establishing the tools, we apply them in two dimensions to obtain a classification of smooth surfaces and a proof that the mapping class group is generated by Dehn twists. In three dimensions, we will study Heegaard splittings and give a Cerf theoretic proof of the Reidemeister-Singer theorem and of Kirby's theorem. If time remains, we will use our tools to understand 4-manifolds through maps of 4-manifolds to surfaces.

Readings

The class will be patched together from a couple of sources. Below are the books and papers we will follow in the order in which they will appear in our class.

Title	Author
An Introduction to Morse Theory	Matsumoto
A proof of Reidemeister-Singer's theorem by Cerf's methods	Laudenbach
Knots and Links	Rolfsen
Stable mappings and their singularities	Golubitsky, Guillemin
A primer on mapping class groups	Farb, Margalit
A presentation for the mapping class group of a closed orientable surface	Hatcher, Thurston
Functions on Surfaces and constructions of manifolds in dimensions three, four, and five	David Gay

Assessments and Grading

There will be two graded homework assignments as well as a 20 minute presentation at the end of the quarter on a topic related to the course chosen by the student. A list of suggested topics will be provided.

Your grade will be calculated as follows:

Category	Weight
Homework	50%
Final Presentation	50%

Tentative Schedule:

Week	Topics	Assessments	References
Apr 3 – Apr 7	Existence of Morse functions, Morse theory on surfaces, Handle Decompositions.		Matsumoto Chapter 1
Apr 10 – Apr 14	Gradient-like vector fields, Handle cancellation, Handle Rearrangement, Handle Slides.		Matsumoto Chapters 2 and 3. Laudenbach paper.
Apr 17– Apr 21	Heegaard Splittings, Introduction to mapping class groups.		Farb-Margalit Ch 1-3. Rolfsen 9C.
Apr 24– Apr 28	Overview of Cerf theory, Generating the mapping class group.	**Homework 1 Due Apr 24th**	A primer on Mapping class groups Ch 4.
May 1 – May 5	Surgery and handles. Surgery on 3-manifolds. The 3-d cobordism group.		Rolfsen Chapter 9.
May 8 – May 12	4-manifolds and Kirby calculus. Some foundations for Cerf Theory		Matsumoto 5.3. Guilleman-Golubitsky Ch. 3, 6.4, 6.5 and 7.6
May 15 – May 19	Presentations for the mapping class group.	**Homework 2 Due May 15th**	Hatcher-Thurston paper
May 22 – May 26	Remarks on 4-manifolds and cobordism groups.		David Gay paper
May 29 – Jun 2	Class Presentations	**Presentations**	See reading list
Jun 5 – Jun 7	Class Presentations	**Presentations**	See Reading list

Resources and Expectations

Academic Integrity: Cheating will be taken very seriously in this course. Familiarize yourself with the code of academic conduct, which can be found at: https://ossja.ucdavis.edu/code-academic-conduct.

Etiquette: There are usually no issues here, but it needs to be said: don't be disruptive and be respectful of your fellow classmates.

Accommodations: Any student with a documented disability who needs to arrange reasonable accommodations must contact the Student Disability Center (SDC). Faculty are authorized to provide only the accommodations requested by the SDC. You can get more information at the SDC website: https://sdc.ucdavis.edu/ or by emailing sdc@ucdavis.edu.