Syllabus: Khovanov Homology and Related Invariants

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* The course website (`https://www.melissa-zhang.com/Teaching/2024-FA/MAT280.html`) contains this syllabus, along with evolving lecture notes, the class calendar, additional materials, and announcements.

Course identifiers

MAT-280 Section 001
CRN 37089
Instructor: Melissa Zhang, MSB 2142

Class Meetings

Lectures: Wellman Hall 235, MWF 1:10 – 2:00 pm
Office hours: MSB 2142, W 3:30-4:30 and R 1-2 (starting the week of Oct 7 per the survey; previously a different date/time)

- The current hours may change after the first week to accommodate the schedules of students taking the course for credit.
- Occasional changes to weekly office hours will be announced here and in class.
- If you need to speak with me privately, please email me with at least 24 hours' notice to schedule an appointment.

Course description

The purpose of this course is to expose graduate students to research-level papers in Khovanov homology and related invariants.

Khovanov homology was the first example of what some call a categorified quantum invariant for links in the three-sphere. In short, it is a combinatorially computed homology-type invariant built from graded vector spaces assigned to all smoothings of a knot diagram. It has been used to prove some important theorems in low-dimensional topology, and has been extended, generalized, and related to invariants in many areas in topology.

In this course, we survey the variations of Khovanov homology (Kh), including for instance Bar-Natan's cobordism category, equivariant Kh, symplectic Kh, Kh's relation with Heegaard Floer homology, and potentially also harder constructions, such as but not limited to Lipshitz-Sarkar Khovanov stable homotopy type or various geometric constructions via derived categories of coherent sheaves.

Prerequisites include working familiarity with chain complexes and homology. All necessarily topological background will be discussed in class, and additional tools in homological algebra, homotopy theory, and algebraic geometry will also be introduced in class as needed.

Assignments and Grading

There are three main components of this course along with their weight in the computation of the overall grade for the course:

1. Lectures (30%):
    - Attending lectures and staying connected with the course material is the core of this course. There are 30 total lecture days.
    - I may sometimes need to hold some lectures online or invite guest lecturers when I am out of town. At present, I know that I will be away during the last lecture, on December 6, but will likely be able to teach virtually.
2. Exercises (30%): **Weekly exercises are due Sunday nights at 11:59 pm on Canvas.**
    - Exercises provided within the lecture notes will range in difficulty, and will cover both computations and more theoretical proofs. I will be expecting you to have looked at and attempted all the exercises, and I will assume this during lectures.
    - Students will be responsible for turning in solutions to **two exercises each week** for 10 total weeks, but can choose which problems to attempt. The chosen exercises should ideally be relevant to the most recent material, but if you are falling behind, I would rather you make sure you do the exercises marked "important" that you have yet to try.
    - Grading will be partially based on honest effort, which will be determined by the write-up submitted online to Canvas. Homework write-ups may be handwritten or typed, but in any case are required to be neat.
    - The weekly due date for these problems will be decided on the first day of class. There is no late policy; you will be graded on whatever you have submitted on the weekly due date.
3. Project (40%):
    - During the latter half of the quarter, each student will independently study a topic we haven't covered in class or only touched upon, and prepare a 5-10 page expository article on the topic (typeset, using a standard amsart template).
    - More details will be available later in the quarter, including a rubric including criteria such as mathematical depth, originality, nontriviality of examples, and quality of writing.
    - Optional drafts of the article will be due
        (a) before Thanksgiving on Sunday, November 24 and
        (b) just before finals week, on Wednesday, December 4.

        These will not impact your grade and are for accountability and feedback only. The polished, final article will be due during finals week, on **Thursday, December 12, at 11:59 pm.**
    - After the quarter, I would like to post your articles here on this class webpage, but only with your permission. The reason for this is two-fold:

  - You may dig deeper into a topic that I have yet to learn, with your particular expertise. Your notes would then supplement my lecture notes for anyone trying to learn about Khovanov homology.
  - Knowing that your article might be read by others may force you to write your exposition more clearly, and to learn the material more deeply.

Final letter grades will be determined by the distribution of numerical grades at the end of the quarter.

Accessibility

For accommodations for disabilities, go to `https://sdc.ucdavis.edu` and begin the process as soon as possible. I will need to approve a letter from the Student Disability Center before making any accommodating changes to the policies stated on this syllabus for you. It is the student's responsibility to make sure all accommodations are set up through the SDC ahead of exams or class meetings where accommodations are needed. Please email me if you have any questions.

Policies

As this is a graduate course, I am assuming that you are familiar with the university-wide policies on academic integrity, plagiarism, harassment, and other foundational aspects of our social contract. Here are the main points:

- **Behave respectfully toward everyone.** In this classroom, you will be treated with respect, and I welcome individuals of all ages, backgrounds, beliefs, ethnicities, genders, gender identities, gender expressions, national origins, religious affiliations, sexual orientations, ability, and other visible and non-visible differences. All members of this class are expected to contribute to a respectful, welcoming and inclusive environment for every other member of the class. (Source: modified from `https://docs.asee.org/public/LGBTQ/Diversity_Statement.pdf`)

- **Don't plagiarize.** You will get out of this class as much as you put in, and plagiarizing will not help you grow as a mathematician. You are free to and encouraged to collaborate with other students in the class, and I am fine with you seeking help from other people or the internet as needed for you to be able to learn the material deeply and also efficiently. But the solution you write down must be in your own words, and reflect your understanding of the material.

  The reason for this particular policy is that, when you're doing research-level mathematics, it is important to be able to make use of all the materials available to you, including websites like MathOverflow or StackExchange. Just remember that the goal is to help you develop into a research-level mathematician who is able to learn diffcult topics and solve previously unsolved problems. You have to learn for yourself how long you should attempt a problem before looking for help, keeping in mind that, when you're doing original research, you will only be able to find potential hints out in the world, but will ultimately have to find the full solution yourself.