FINAL EXAMINATION

21A §E01-07, 3:30-5:30 pm
Monday December 11, 2017

Declaration of honesty: I, the undersigned, do hereby swear to uphold the very highest standards of academic honesty, including, but not limited to, submitting work that is original, my own and unaided by notes, peeking at the person next to me whose answer is probably wrong anyway, books, calculators, mobile phones, blackberries, blueberries, boysenberries, raspberries, artificial intelligence or any other electronic device. Volcanic-emotional-support-pet rocks without tattoos permitted.

Well-organized and explained responses will receive more credit.

Signature _________________  Date _______________
Q1 scratch/extra space (do not erase your scratch computations, they might earn partial credit):
Question 1

Define what the symbols

\[ \lim_{x \to a^+} f(x) = \infty \]

mean (include a picture in your answer). Use your definition to prove that

\[ \lim_{x \to 0^+} \frac{1}{x} = \infty . \]
Q2 scratch/extra space (do not erase your scratch computations, they might earn partial credit):
Question 2 Calculate city! Compute the following quantities (do show your work):

(i) \( \lim_{x \to \infty} \frac{x^2 + 2x + 2}{x^2 + 2x + 1} \)

(ii) \( d(x^x) \)

(iii) \( \lim_{x \to 0} \frac{\sqrt{1 + x} - 1}{x} \)

(iv) \( \frac{d}{dx} \left( \frac{x(x + 1)(x + 2)(x + 3)(x + 4)(x + 5)(x + 6)(x + 7)(x + 8)}{x} \right) \)
Q3 scratch/extra space (do not erase your scratch computations, they might earn partial credit):
Question 3
A 4’ tall turkey stands 2’ away from the base of a lamppost at night and casts a 1’ shadow. The turkey then saunters\(^1\) away from the lamppost at 1’/second. At what rate does the turkey’s shadow lengthen?

\(^{1}\text{This means to walk in a relaxed fashion!}\)
Q4 scratch/extra space (do not erase your scratch computations, they might earn partial credit):
Question 4
Use the Newton–Raphson method to compute an approximation\(^{2}\) for \(\sqrt[3]{3}\).

\(^{2}\text{Note: }144^3 = 2985984 \& 145^3 = 3048625; \text{ a well-explained result accurate to 2 decimal places will earn full credit for this problem.}\)
Q5 scratch/extra space (do not erase your scratch computations, they might earn partial credit):
Question 5
Let $n$ be a positive integer, and $x$ and $y$ be positive numbers that obey

$$x + y = 1.$$

Find the values of $x$ and $y$ that minimize $x^n + y^n$.

\footnote{You might be able to guess the answer using a symmetry argument. To earn full credit, use calculus to show that your guess is correct.}
Q6 scratch/extra space (do not erase your scratch computations, they might earn partial credit):
Question 6

Let \( f : \mathbb{R} \to \mathbb{R} \) where

\[ f(x) = \frac{1}{1 + x^2}. \]

Sketch the curve \( y = f(x) \). The better your sketch, the more credit you will earn.