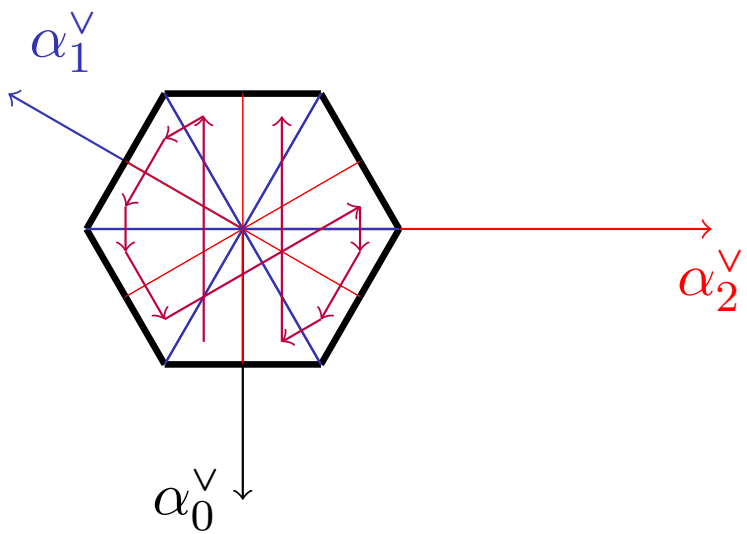
Impact of computer assisted experimentation in combinatorics

Anne Schilling, UC Davis

Sage Days, Ljubljana, July 8, 2019



A Story

One of my passions are

crystal bases which provide a combinatorial tool to study algebraic/geometric structures such as

- quantum groups
- affine Schubert calculus
- symmetric functions
- representation theory

Combinatorics lends itself to computational analysis!

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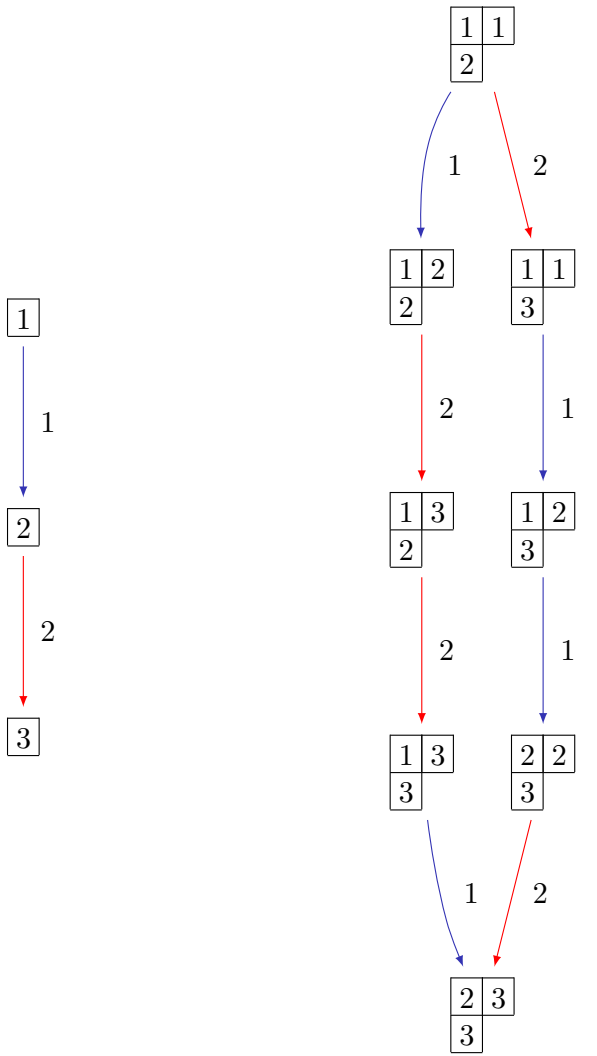
Crystals

$B(\square)$ $B(\ydiagram{2,1})$



Lonely Programming ...

Before 2008:

- Programmed crystals in Mathematica
- Programmed what I needed right then for research
- No tests or documentation
- Could not reuse my own code a few weeks later (forgot how it worked ...)
- Kept writing similar code over and over

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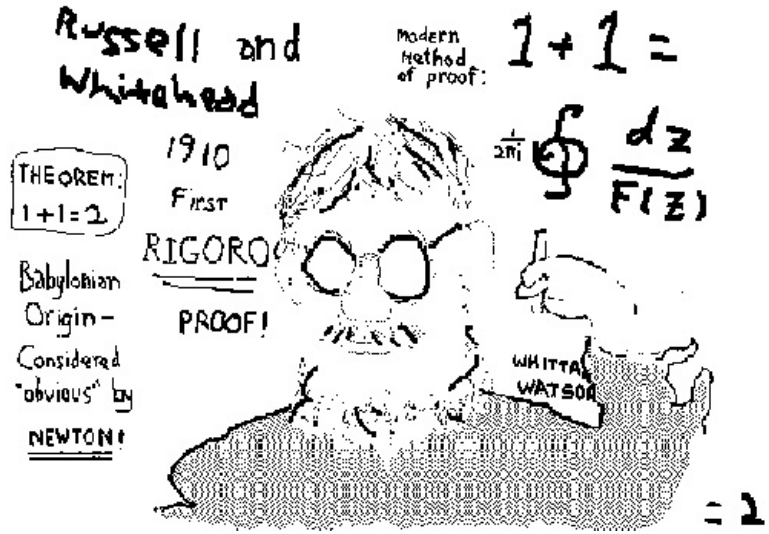
Sage Days 7 at IPAM in 2008

with Nicolas Thiéry
started porting crystal code to Sage



Dan Bump
uses crystals in number theory

What can SageMath do?

```
sage: B = crystals.Tableaux(['A',2],shape=[2,1])
sage: u = B.highest_weight_vector(); u
[[1, 1], [2]]
sage: b = u.f(1); b
[[1, 2], [2]]
sage: type(b)
<class 'sage.combinat.crystals.tensor_product.
CrystalOfTableaux_with_category.element_class'>
sage: u.weight()
(2, 1, 0)
sage: b.weight()
(1, 2, 0)
```

Implementation of a crystal

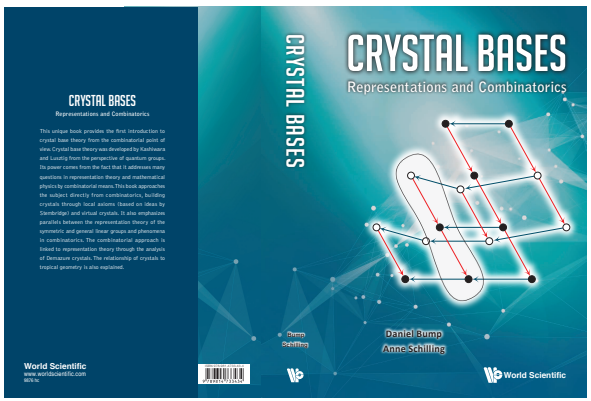
```
class HighestWeightCrystalOfTypeA(UniqueRepresentation, Parent):

    def __init__(self, n = 3):
        Parent.__init__(self, category = ClassicalCrystals())
        self.n = n
        self._cartan_type = CartanType(['A',n])
        self.module_generators = [ self(1) ]

    def _repr_(self):
        return "Highest weight crystal of type A_%s
                    of highest weight omega_1"%(self.n)

    class Element(ElementWrapper):

        def e(self, i):
            if self.value == i+1:
                return self.parent()(self.value-1)
            else:
                return None

        def f(self, i):
            if self.value == i:
                return self.parent()(self.value+1)
            else:
                return None
```

Moral of the Story ...

End/beginning of the Story ...

Semester long program at ICERM on
Automorphic Forms, Combinatorial Representation Theory and Multiple Dirichlet Series, Spring 2013

Thematic Tutorial: Lie Methods and Related Combinatorics in Sage



k-Schur functions and affine Schubert calculus

Active tickets! Active tickets!

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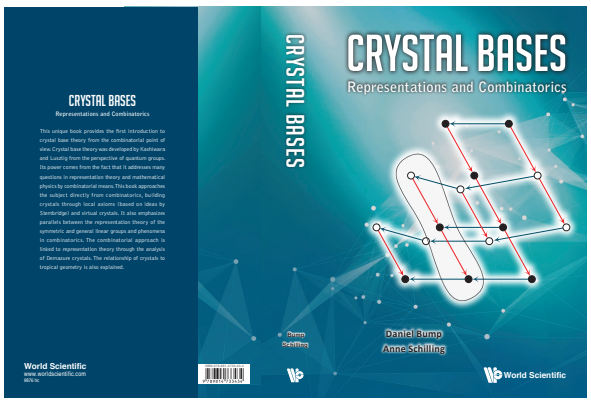

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SageMath

SageMath's mission: **Creating a viable free open source alternative to Magma™, Maple™, Mathematica™, and Matlab™.**

- free
- open source
- anyone can contribute!

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History:

- SageMath project began by William Stein in 2005
  SAGE="Software for Arithmetic Geometry Experimentation"
- Quickly expanded beyond number theory; attracted more users, developers, funding
- sagenb.org now has over 90,000 accounts

Sage-combinat: **"To improve the open source mathematical system** SageMath **as an extensible toolbox for computer exploration in (algebraic) combinatorics, and foster code sharing between researchers in this area."**

Try it!

- CoCal https://cloud.sagemath.com
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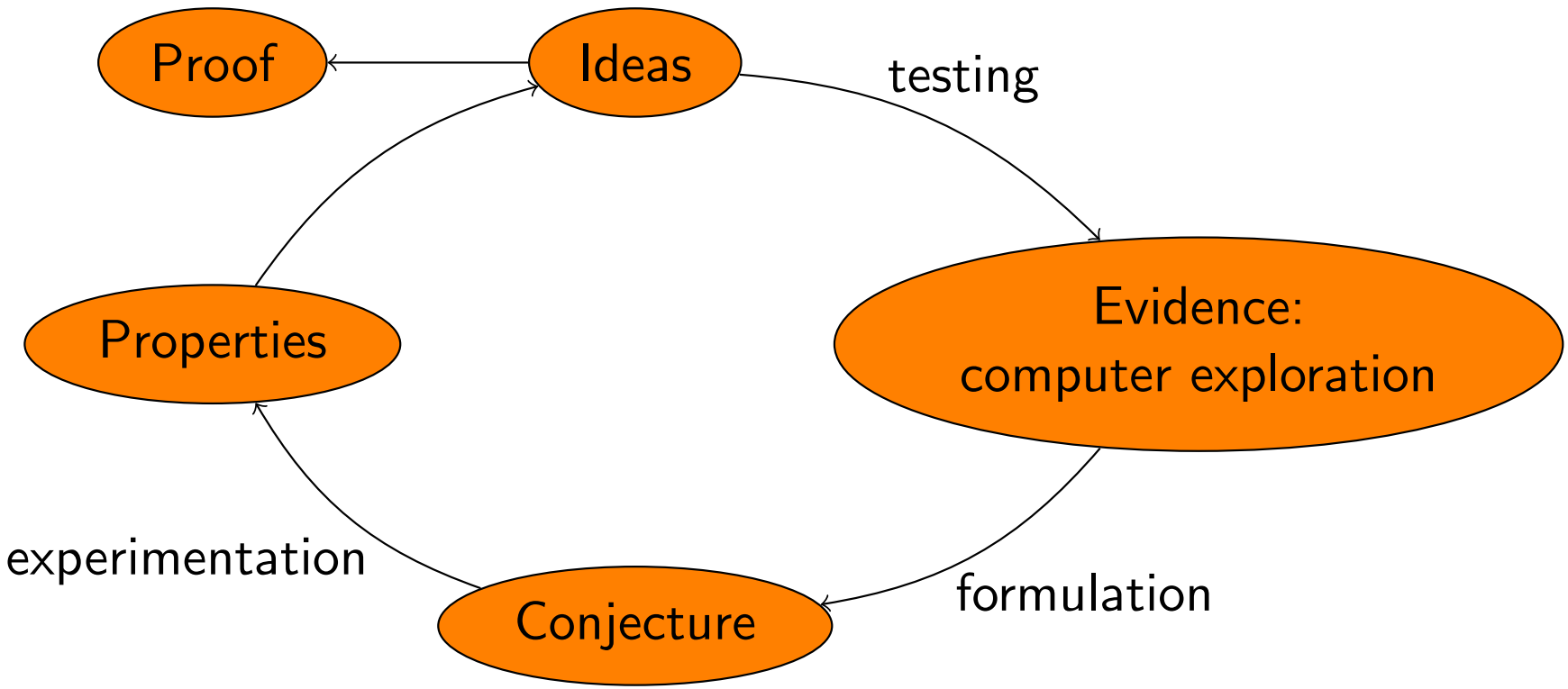
Experimental Mathematics



Set-valued and multiset tableaux

- Set-valued tableaux: order $\max(A) \leqslant \min(B)$ in row AB
  $\max(A) < \min(C)$ in column $\begin{matrix} C \\ A \end{matrix}$
  Monical, Pechenik, Scrimshaw arXiv:1807.03294

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234	4	
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Possible projects

- Classes for various tableaux with various orders
- Insertion algorithms associated to these tableaux (0-Hecke insertion, RSK into various classes,....)
- Crystal structure on these tableaux
- Symmetric chain decompositions of posets

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