Math 127C Practice Midterm II Spring 2024

1. (Torsion) Consider the function

$$f((x,y,z)) = \begin{bmatrix} y\cos(x) \\ z\cos(y) \\ 2x\cos(z) \end{bmatrix}.$$

Find

$$(f \circ f \circ f)' \left((0,0,0); \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right).$$

2. (IFT) Consider the function

$$f((x,y)) = \begin{bmatrix} x^2\sin(y) \\ x^2\cos(y) \end{bmatrix}.$$

Find a point in the plane which has a neighborhood on which f is invertible. Call this inverse function g. Choose a point (a,b) and find $(Dg)(a,b)$.

3. (Absolute) Show that if $f : Q \to \mathbb{R}$ is any bounded function on a rectangle then

$$\overline{\int_Q} f - \underline{\int_Q} f \geq \overline{\int_Q} |f| - \underline{\int_Q} |f|.$$

Use this to show that if f is integrable over Q then so is $|f|$.

4. (Ideal) Assume that $S \subseteq \mathbb{R}^n$ is bounded while $f : S \to \mathbb{R}$ and $g : S \to \mathbb{R}$ are both bounded and both continuous. Write $fg : S \to \mathbb{R}$ for the pointwise product function with $(fg)(x) = f(x)g(x)$. Show that if f is integrable over S then fg is also.