

Math 21C Practice Final Winter 2018 Practice Final, March 22.
To receive full credit you must show all of your work. Please do not use any cell phones, notes or books. A calculator is allowed. Do not simplify your solutions

1. Determine whether the following limit exists.

$$\lim_{n \rightarrow \infty} \frac{10^{6n}}{\sqrt{n^n}}$$

2. How many terms of the following convergent series should be used to estimate its value with an error of at most 0.1?

$$\sum_{n=1}^{\infty} 2ne^{-n^2}$$

3. Use the alternating series error bound to estimate the error from using the first two nonzero terms in the Maclaurin series for e^{-x^2} to estimate the definite integral

$$\int_0^{\frac{1}{10}} e^{-x^2} dx.$$

4. Determine whether the following series converges.

$$\sum_{n=1}^{\infty} \frac{1}{\ln(n)\sqrt{n}}$$

5. Determine the values of x for which the following series converges. Be sure to check the end points of the interval.

$$\sum_{n=0}^{\infty} \frac{(2-x)^n}{\sqrt{n}}$$

6. Find the first three nonzero terms of the Taylor series about $x = 0$ for the following function.

$$\frac{1+x^2}{1-2x}$$

7. Find an equation for the set of points in space which are the same distance from the origin as they are from the point $(1, 1, 1)$.
8. Find the cosine of any one of the angles in the triangle with corners at $(1, 1, 1)$, $(1, 2, 3)$ and $(5, 4, 3)$.

9. Find an equation for the plane containing the points $(1, 1, 1)$, $(1, 2, 3)$ and $(5, 4, 3)$.
10. If the distance between the points P and Q is 5 and the distance between the point P and the line $Q + t\mathbf{u}$ is 4 what is the distance between the point P and the plane through Q and perpendicular to \mathbf{u} ?
11. Show that the following limit does not exist.

$$\lim_{(x,y) \rightarrow (1,0)} \frac{xy - y}{(x - 1)^2 + y^2}$$

12. Find $\frac{dx}{ds}$ at $s = \frac{\pi}{2}$ if $w = x^2y^2$, $y(s) = \sin(s)$, $x(s)$ is unknown but $x(1) = 3$ and $\frac{dw}{ds} = 2$ at $s = 1$.
13. Find the two directions in which the directional derivative of $f(x, y)$ at the point $(1, 1)$ is zero.

$$f(x, y) = x^2y - y^2x$$

14. Find the point on the elliptic paraboloid given by $z = 1 - x^2 - y^2$ which is closest to the point $(0, 0, 2)$.
15. Find and classify the critical point(s) and find the maximum value of the function $y^2 - x^2$ on the circular disk given by $4(x - 1)^2 + 4y^2 \leq 9$.
16. Find the largest area of a rectangle with corners on the ellipse $\frac{x^2}{16} + y^2 = 1$ and edges parallel to the x and y axes.
17. (Optional extra credit problem.) Find the value of a for which the following limit exists and find the limit.

$$\lim_{x \rightarrow 0} \frac{\sin(ax) - \sin(x) - x}{x^3}$$